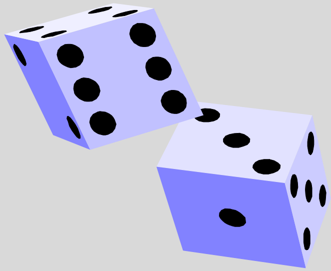
ANNEX: PROBABILITY BASIC CONCEPTS APPLIED TO PATTERN RECOGNITION

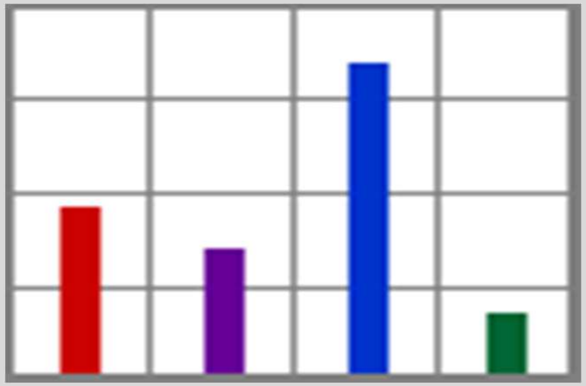
Grado en Ingeniería Informática

Curso 2014 / 15

Topics

1. Concepts
2. Classical probability
3. Frequently discrete distributions
4. Frequently continuous distributions





CONCEPTS

Probability concept

- **PROBABILITY**: is the scientific discipline that studies the laws of chance.

  - ¿Cómo osamos hablar de leyes del azar? ¿No es, acaso, el azar la antítesis de cualquier ley?

    Bertrand Russell

  - Es un hecho destacable que una ciencia que empezó analizando juegos de azar acabe convirtiéndose en el más importante objeto del conocimiento humano.

    Pierre Simon Laplace

Probability concept

- Simple problems:
  - Remove cards from a deck
  - Throwing a coin
  - Throw a dice

- Complex problems:
  - Genetics
  - Stock market
  - General Elections
  - Nuclear physics
  - Test scores of some subjects
  - ALMOST ANY PROBLEM OF NATURE

Random experiment

- Anything that conducted under the same conditions, provide an impossible result to predict a priori.
- For example:
  - Throw a dice
  - Draw a card from a deck
  - A coin is tossed. If heads, a ball is drawn from a U1 urn with a given composition of colored balls and if tails, a ball from a U2 urn, with another given composition of colored balls, is drawn.

Random experiment

- An experiment is random if the following conditions are met:
  - It can be repeated indefinitely, always in the same conditions;
  - Before you realize it, you can not predict the results to be obtained;
  - The result obtained, ***e***, belongs to a previously known set of possible outcomes. This possible outcomes set, we will call the sample space.

Sample space

- It is the possible outcomes collection of the experiment
- EXAMPLES:
  - Flipping a coin and observe the results:
    E = {HEADS (H) TAILS (T)}
  - Throw a dice:
    E = {1, 2, 3, 4, 5, 6}
  - Throwing two coins
    E = {HH, HT, TH, TT}

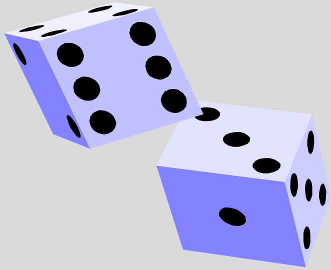
Sample space

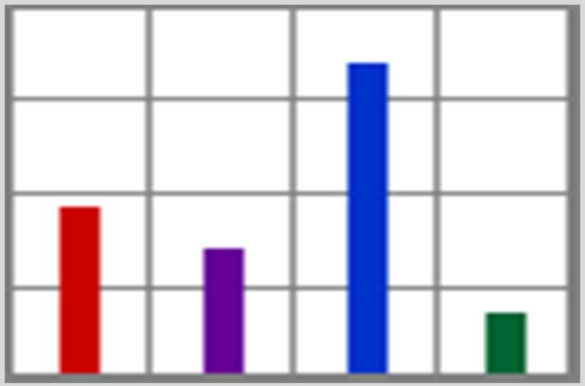
- Experiment: Throwing two dice

E = (1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6)
(2,1) , (2,2) , (2,3) , (2,4) , (2,5) , (2,6)
(3,1) , (3,2) , (3,3) , (3,4) , (3,5) , (3,6)
(4,1) , (4,2) , (4,3) , (4,4) , (4,5) , (4,6)
(5,1) , (5,2) , (5,3) , (5,4) , (5,5) , (5,6)
(6,1) , (6,2) , (6,3) , (6,4) , (6,5) , (6,6)

Event concept

- Event: Any subset of ***E*** (sample space).
- Examples:
  1. When thrown 3 times a coin, the sample space is:
     E = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
  2. The event or event goes "at least two heads" is:
     S = {HHH, HHT, HTH, THH}
  3. The event "at least one tails" is listed:
     S = {HHT, HTH, HTT, THH, THT, TTH, TTT}





CLASSICAL PROBABILITY

Classical probability

- Laplace defines the probability of an event A as:

$$P(A) = \frac{\text{Number of favorable cases}}{\text{Number of possible cases}}$$

- If we throw a dice, what is the probability P(A) of A = *greater than or equal to 5*? And the probability of B = *odd*?
- **Solution:** The six possible cases are equally likely, each has probability 1/6
- P(A) = 2/6 = 1/3 as A = {5,6} has two favorable cases.
- P(B) = 3/6 = 1/2 as B = {1, 3, 5} has three favorable cases

Probability axiom

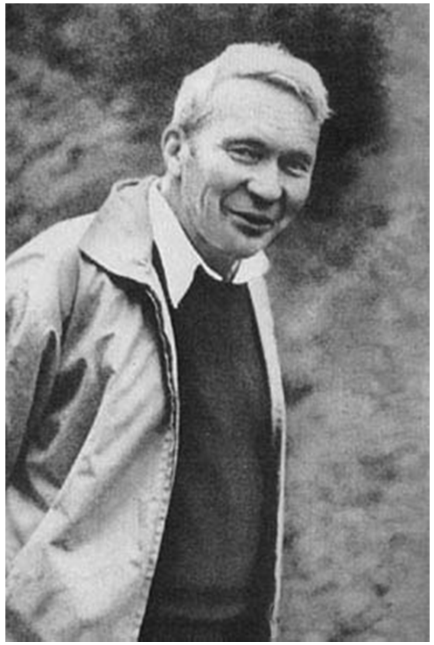
- Probability *P* is called any function that assigns to each event *E* of the sample space to a numerical value *P(A)*, verifying the following axioms:

  (1) No negative: $0 \leq P(A)$

  (2) Normalization: $P(E) = 1$

  (3) Additivity: $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \emptyset$

(where Ø is the empty set).



Kolmogorov, 1933

Probability function

- Probability function assigns each variable value corresponding probability.
- In the experiment, "*Throwing a dice*", the probability function *f(k)* is:

X	1	2	3	4	5	6
f(k)=P(X=k)	1/6	1/6	1/6	1/6	1/6	1/6

Probability function

- In the experiment, "*Throwing a loaded dice*," the probability function *f(k)* might be:

X	1	2	3	4	5	6
f(k)=P(X=k)	1.5/6	1/6	1/6	1/6	1/6	0.5/6

Continuous case

- **Problem**: what if the events are not a discrete variable, but continuous?
- Examples:
  - Take a fish out of water, and measure its length
  - The quotation on the stock value
  - The time spent in a 1500m race.
- In this case, there would be infinite possible cases?
- To solve the problem, we introduce a new concept

Probability distribution function

- Given a random variable *X* the probability distribution function *F(X)* assigned to *X* defined on a probability event.

$$F(a) = P(X \leq a)$$

- *F* must satisfy that:
  - *F* must be continuous and monotone increasing
  - $\lim_{x \to -\infty} F(x) = 0$
  - $\lim_{x \to \infty} F(x) = 1$

Probability distribution function

- Discrete case

$$F(x) = \sum_{t=-\infty}^{x} f(t)$$

- Continuous case

$$F(x) = \int_{t=-\infty}^{x} f(t)$$

Probability density function

- Mathematically, the probability density function is the derivative of *F*
- The probability density function should meet f(x) >= 0, and that the integral of f in [- ∞, ∞] is 1

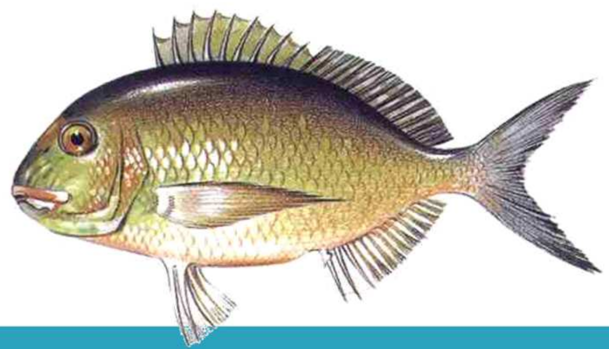
$$F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x) \cdot dx$$

$$f(x) \geq 0$$

$$\int_{-\infty}^{+\infty} f(x) dx = 1$$

Example: the golden

- It has a high and compact body.
- Commonly called to present a yellow band on the front of the head and between the eyes.
- Its back is silver-gray, yellow-gray flanks with some golden shimmer, also presented to the height of the gill opening a dark spot.
- It can grow to 70 cm, and its most common size 20-50 cms.



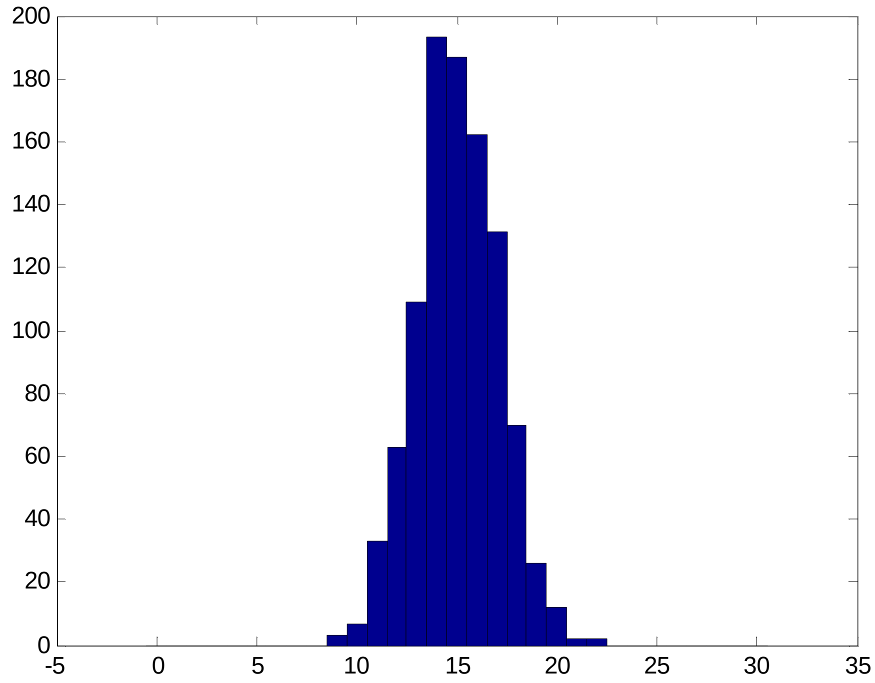
Gold distribution (discrete variable)

```
x=floor(15+2*randn(1,1000));
```

x= [11 16 13 15 11 18 18 18
15 14 17 14 19 15 17 12
16 11 15 13 15 13 15 14
12 18 16 15 14 …

Histogram concept (discrete variable)

```
hist(x,0:30)
s=hist(x,0:30)
bin = 12; length(find(x==bin))
```



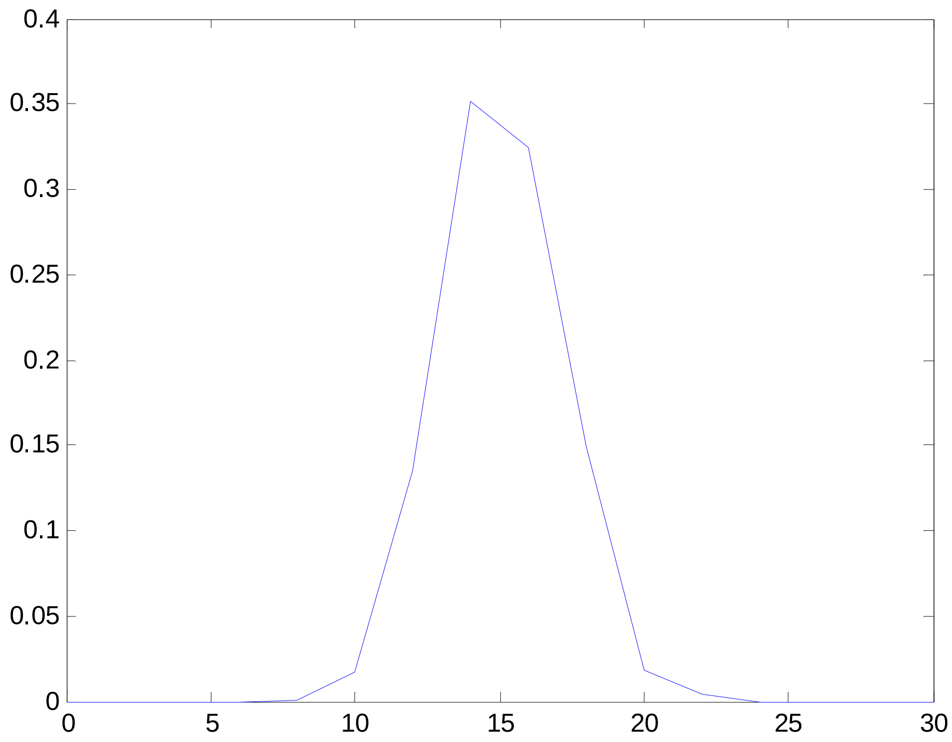
Question

- How can we turn the histogram on a probability function?

Probability concept (discrete variable)

```
s = hist(x,0:2:30)
f = s / length(x)
plot(0:2:30,f)
```

f(X) ≈ P(x==X)




```
F = cumsum(f)
plot(0:2:30,F)
```

F(X) ≈ P(x<=X)

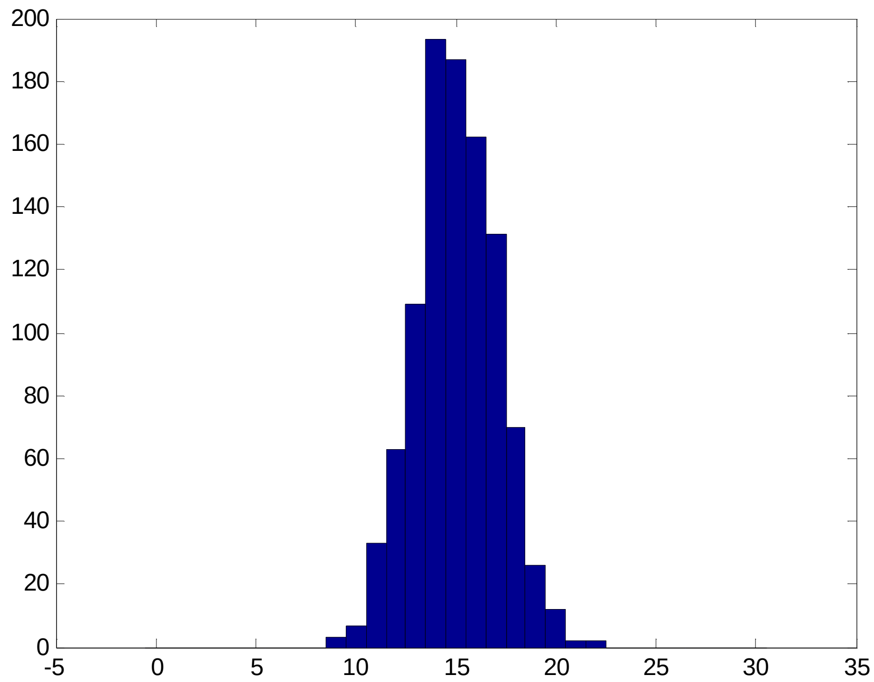
Golden distribution (continuous variable)

x=~~floor~~(15+2*randn(1,1000));

x= [	11.21	16.14	13.18	15.16	11.22
	16.19	11.21	15.14	13.07	15.09
	12.14	18.92	16.81	15.27	…

Histogram concept (continuous variable

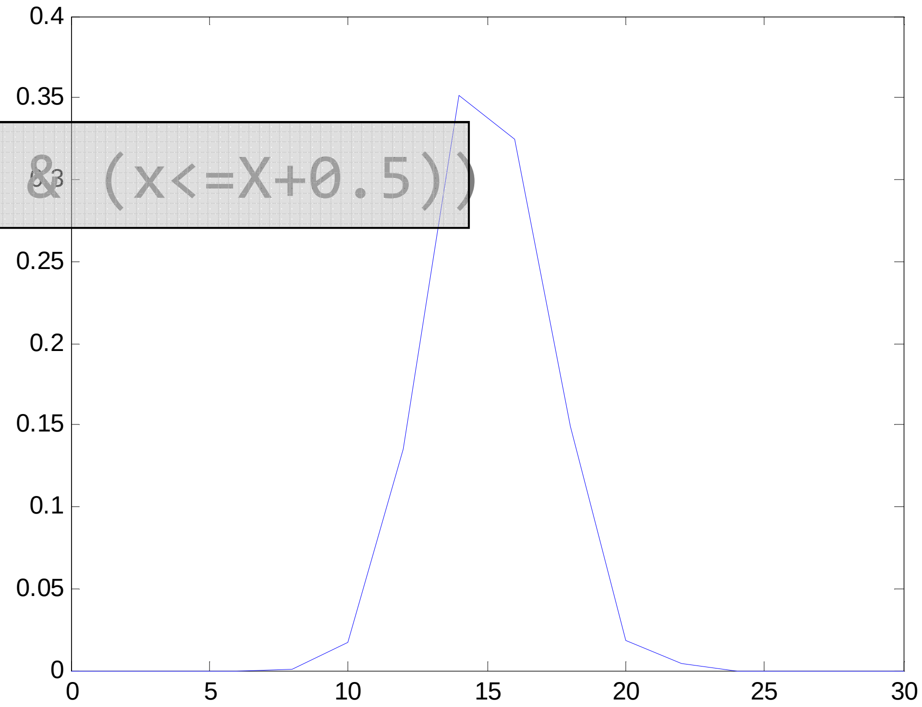
```
hist(x,0:30)
s=hist(x,0:30)
bin = 10; length(find((x>bin-0.5) & (x<bin+0.5)))
```



Probability concept (continuous variable

```
s = hist(x,0:2:30)
f = s / length(x)
plot(0:2:30,f)
```

```
f(X) ≈ P( (x>=X-0.5) & (x<=X+0.5))
```

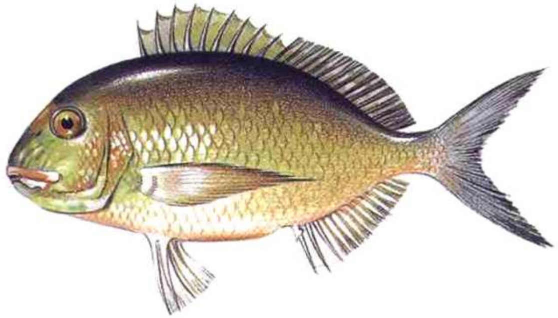



```
F = cumsum(f)
plot(0:2:30,F)
```

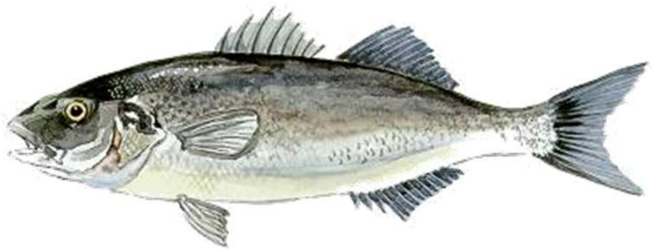
```
F(X) ≈ P(x<=X)
```

Problem: classify fish by its length

```
x=15+2*randn(1,1000);
plot(x,0,'.b','MarkerSize',5)
hold on
```

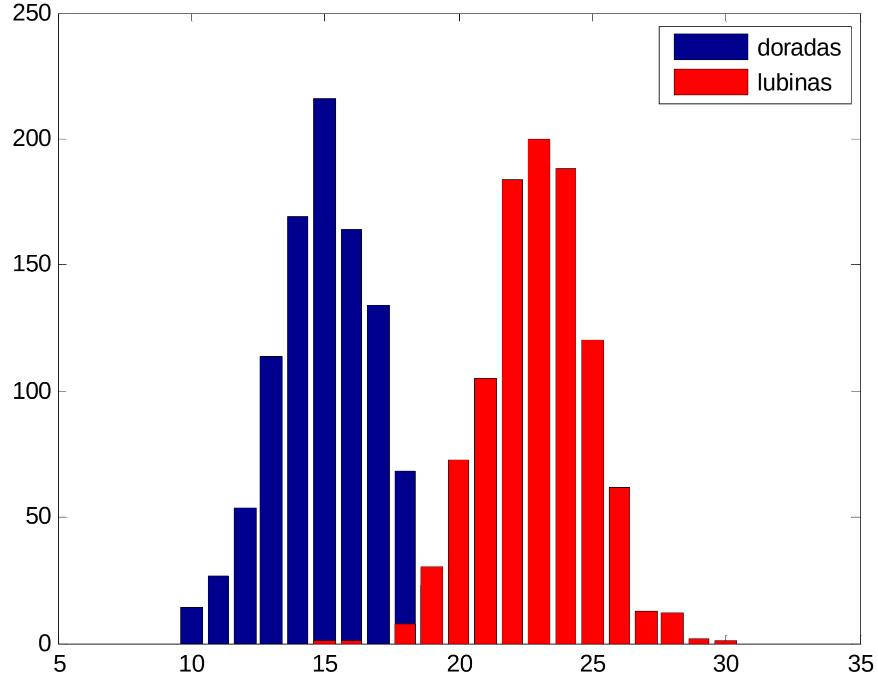


```
y=23+2*randn(1,1000);
plot(y,1,'.r','MarkerSize',5)
axis([0 40 -10 10])
```



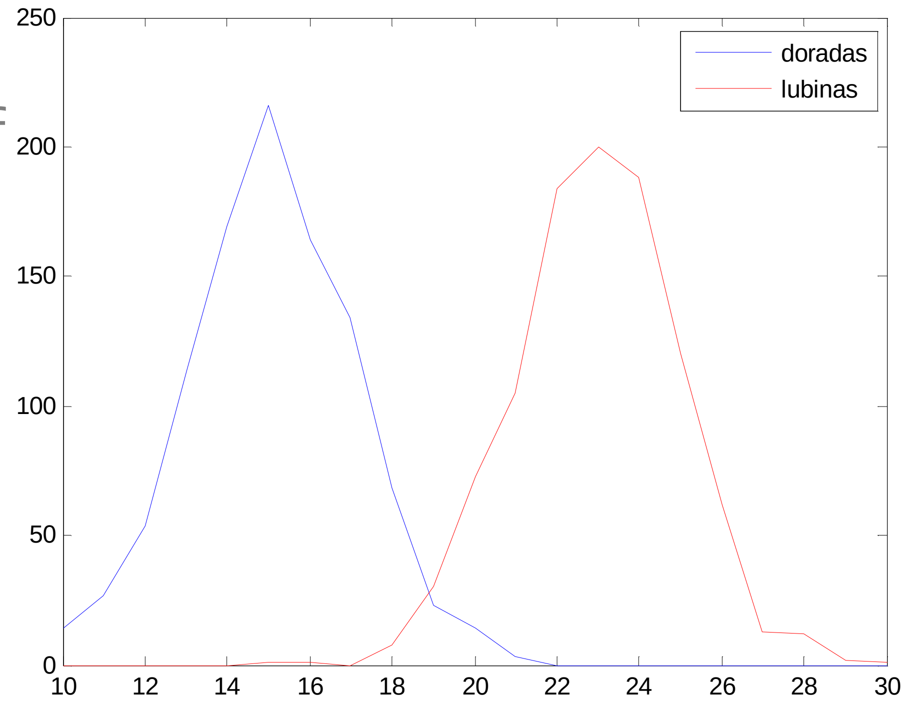
Both species histograms

```
interv=10:30;
vx=hist(x,interv);
vy=hist(y,interv);
bar(interv,vx);hold on;
bar(interv,vy,'r');hold off
legend('doradas','lubinas')
```



Both species histograms

```
interv=10:30;
vx=hist(x,interv);
vy=hist(y,interv);
plot(interv,vx);hold on;
plot(interv,vy,'r');hold off
legend('doradas','lubinas')
```

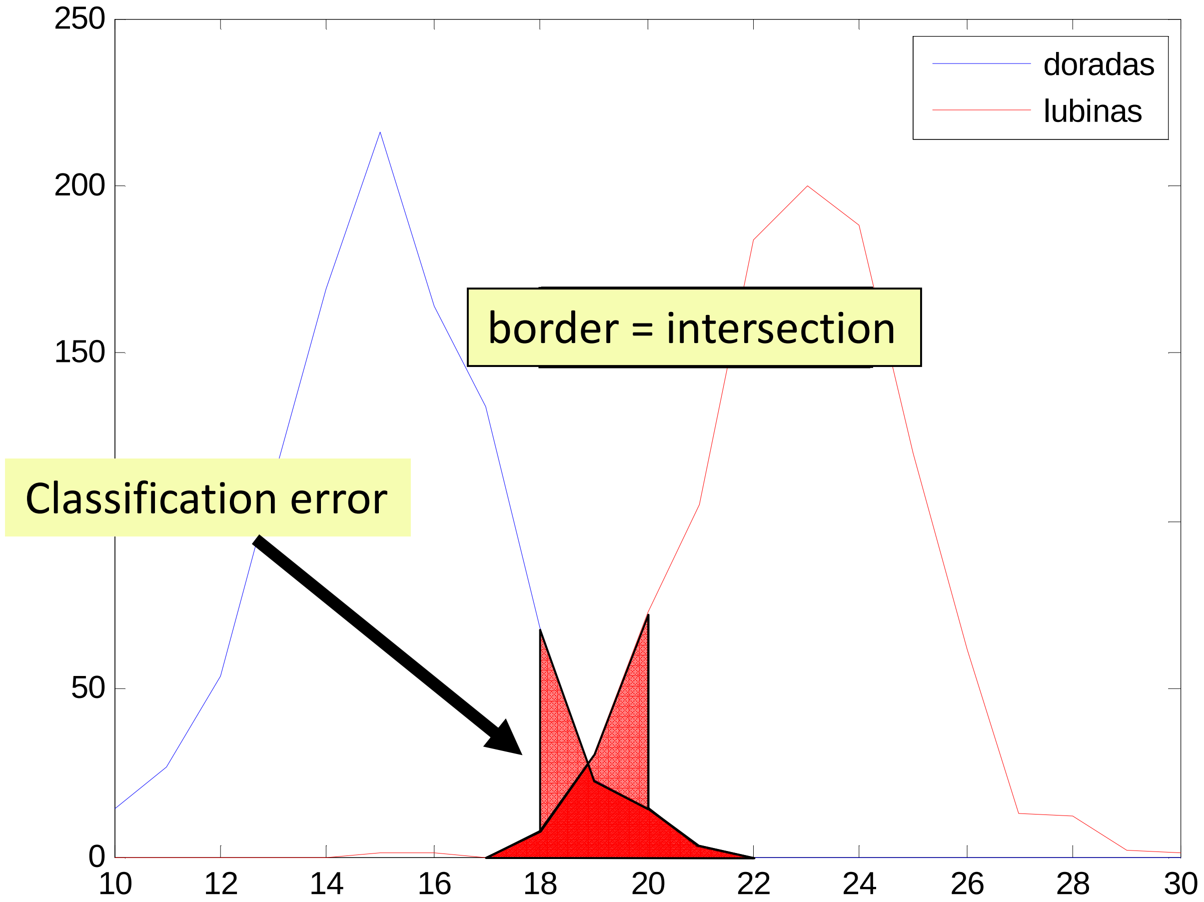


Question

- If we choose a fish at random and measured, and its length is 15.23 cm. Do you know what species it is?
- What if measured 19.54 cm. ?
- At what point do we say that value is more likely to be a golden that a sea bass?

THIS IS A CLASSIFICATION SYSTEM!!

Optimal value = minimun error



de Patrones

Numerical result

```
clc
x=15+2*randn(1,1000);
y=23+2*randn(1,1000);
for Frontera = 17:21
    Err1 = length(find(x>Frontera));
    Err2 = length(find(y<Frontera));
    ErrTotal = Err1 + Err2;
    disp([' Frontera = ' num2str(Frontera)])
    disp(['   Err1 Err2 ErrTot'])
    disp([Err1 Err2 ErrTotal])
end
```

Exact value

- In this case, because distributions are equal, symmetric, and the number of data of each species is the same, it can be shown that the true value is:

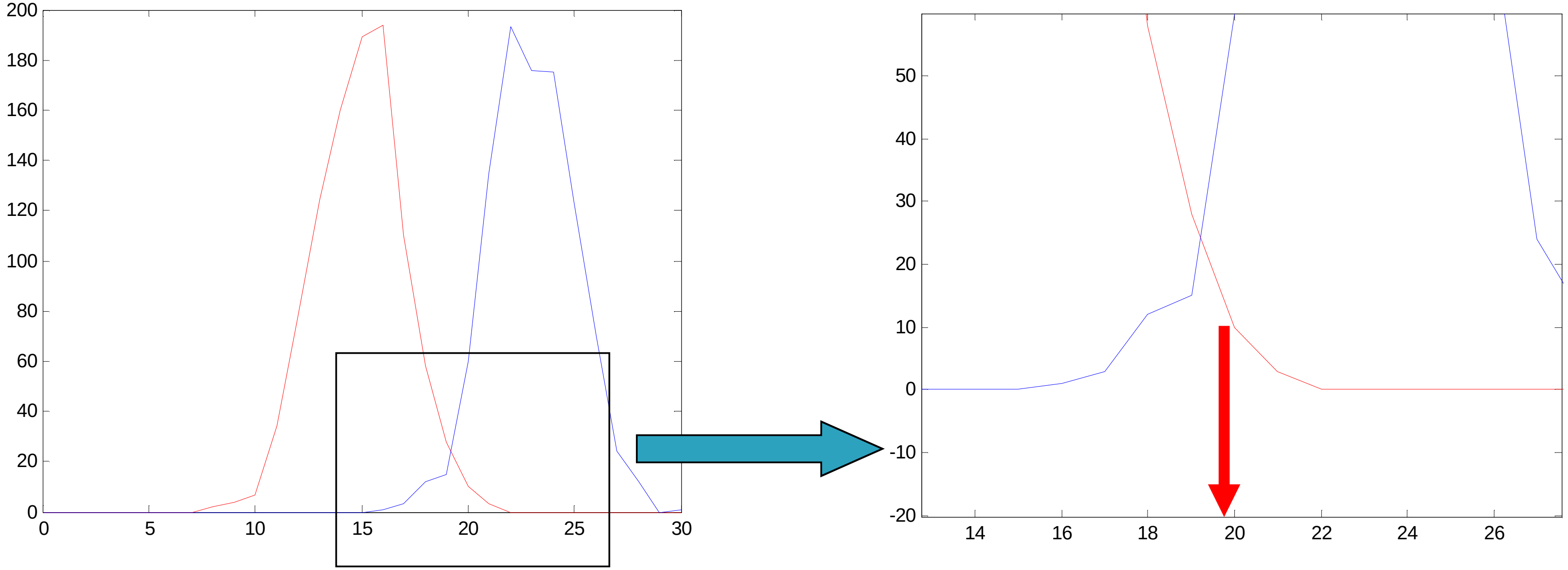
Border = 19 cm.

- Normally this is not the case
- What if I do not know the exact solution?

Solution 1

- Draw histograms of both species, and find the cutoff

```
sx=hist(x,0:30);plot(0:30,sx,'r');hold on;
sy=hist(y,0:30);plot(0:30,sy, 'b');hold off;
```



Solution 1 problems

- Inaccuracy due to discretization on
  - The narrower each histogram bar is more accurately
  - But I have less value per bar
  - COMMITMENT

I need to get all the fish of the sea for the exact value of the optimal frontier !!

Solution 2

- We assume that the distributions are symmetrical and equal in form
  - mx = mean(x)
  - my = mean(y)
  - Frontera = (mx+my) /2

- QUESTIONS
  - What method works best?
  - What is easier?
  - What is faster to compute?
  - Which method is more rigorous?

Solution 2 problem

- What if the distributions are not symmetrical?

```
x=15+sum(randn(5,1000).^2);
y=23+sum(randn(5,1000).^2);
sx=hist(x,0:40);plot(0:40,sx,'r');hold on;
sy=hist(y,0:40);plot(0:40,sy, 'b');hold off;
title(num2str(0.5*(mean(x)+mean(y))))
```

Solution 2 problem

- What if the distributions are not equal in shape?

```
x=15+2*randn(1,1000);
y=23+4*randn(1,1000);
sx=hist(x,0:40);plot(0:40,sx,'r');hold on;
sy=hist(y,0:40);plot(0:40,sy, 'b');hold off;
title(num2str(0.5*(mean(x)+mean(y))))
```

Solution 2 problem

- What if there is a number of different individuals for each species?

```
x=15+2*randn(1,3000);
y=23+2*randn(1,97000);
sx=hist(x,0:40);plot(0:40,sx,'r');hold on;
sy=hist(y,0:40);plot(0:40,sy, 'b');hold off;
title(num2str(0.5*(mean(x)+mean(y))))
```

General problem

- For everything to work best, we would have to draw and measure **ALL** of sea bream and sea bass
- Because it is impossible to work with a representative **SAMPLE** of the population
- The more items are in my sample, and more representative, the better the estimates

Solution 3

- Take a representative sample of each type of fish
- Best estimate probability density function (shape and parameters) for each type of fish
- Adjust density function to account for the number of individuals in each class
- Find the intersection of two curves analytically and give that value as the optimal boundary

Watch out for special cases

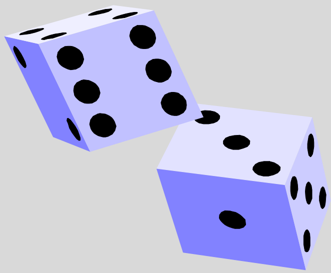
- What happens if the distributions are centered at the same point, but have different widths?

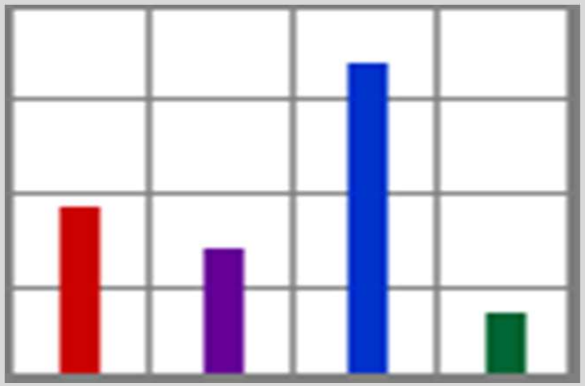
```
x=15+4*randn(1,10000);
y=15+2*randn(1,10000);
sx=hist(x,0:40);plot(0:40,sx,'r');hold on;
sy=hist(y,0:40);plot(0:40,sy, 'b');hold off;
title(num2str(0.5*(mean(x)+mean(y))))
legend('doradas', ‘lubinas')
```

Solution 3 problems

- Best estimate probability density function of a population from a sample
- Best estimate the number of individuals in the population of each class from a sample

!!! MORE STATISTICS !!!





FREQUENTLY DISCRETE DISTRIBUTIONS

Uniform distribution

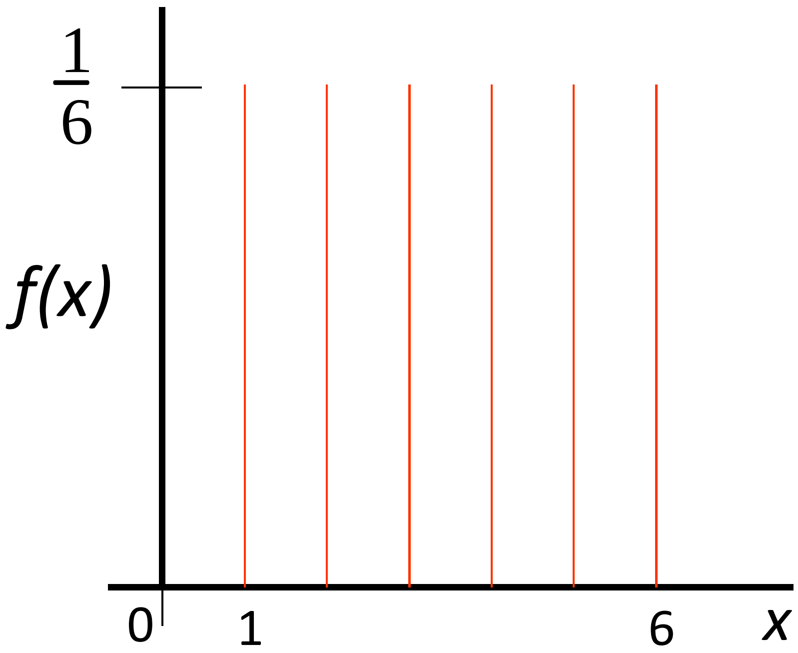
- If all elementary events are equally likely, we can say that *X* is uniformly distributed.
- If the sample space consists of *n* simple events ($0 < n < \infty$), then the discrete probability function is defined as

$$p(x) = 1 / n$$

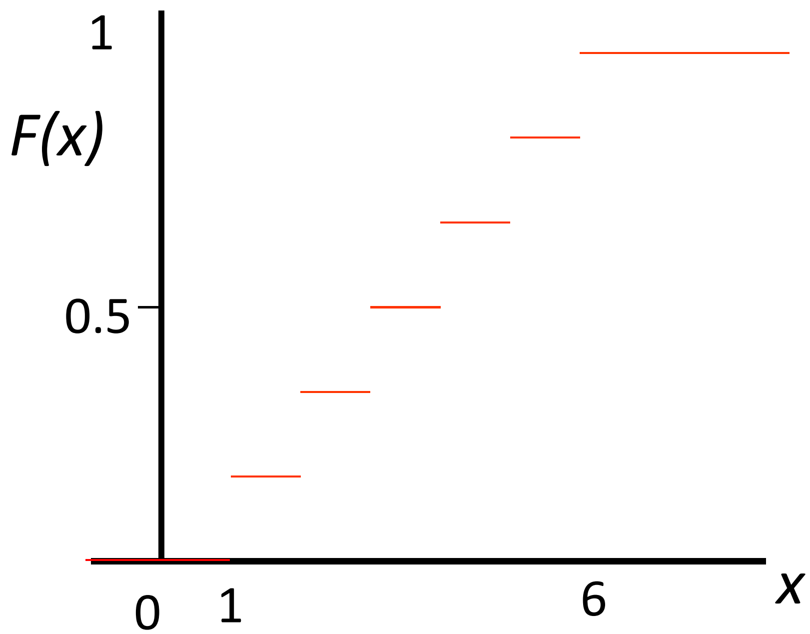
- Examples:
  - Throwing a coin (n = 6)
  - Throwing a dice (n = 2)

Uniform distribution. Example: throwing a dice

- X has the possible values x = 1, 2, 3, 4, 5, 6 once with probability 1/6



Probability function f(x)



Distribution function F(x)

Bernouilli distribution

- The experiment result supports only two outcomes: success (1) or failure (0)
- A typical Bernoulli experiment is throwing a coin with probability *p* for heads and *(1 - p)* to tails
- If the probability of success is *p*:

$$\boxed{f(x) = p^x (1-p)^{1-x} \qquad x = 0,1}$$

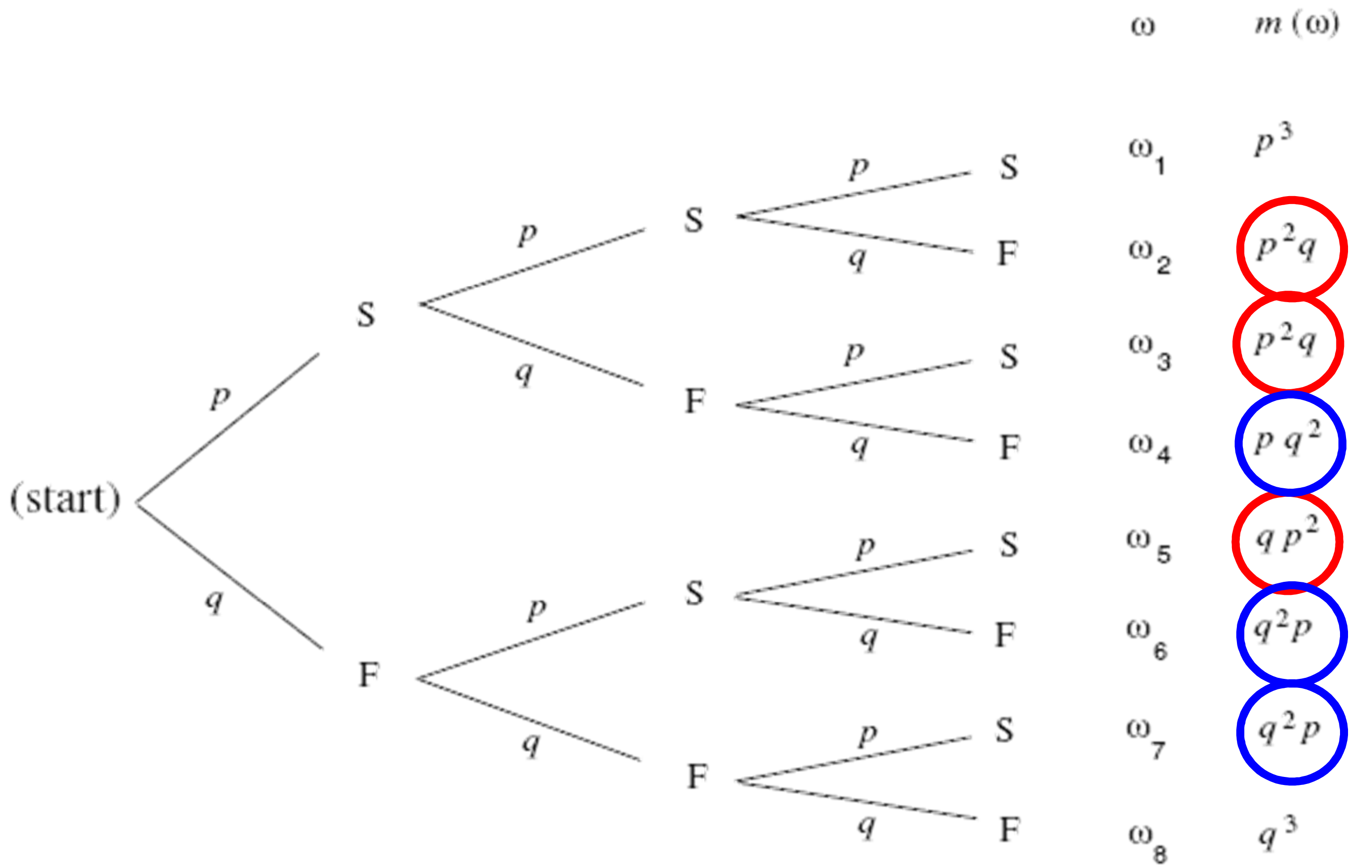
$$\boxed{F(x) = \begin{cases} 1-p, \ \text{para } x = 0 \\ 1, \ \text{para } x = 1 \end{cases}}$$

Binomial distribution

- The binomial distribution appears when we are interested in the number of times that an event A occurs (successes) in n independent trials of a Bernoulli experiment

- Eg .: number of heads in 3 tosses of a coin.

Binomial distribution.No. of faces in 3 pitches

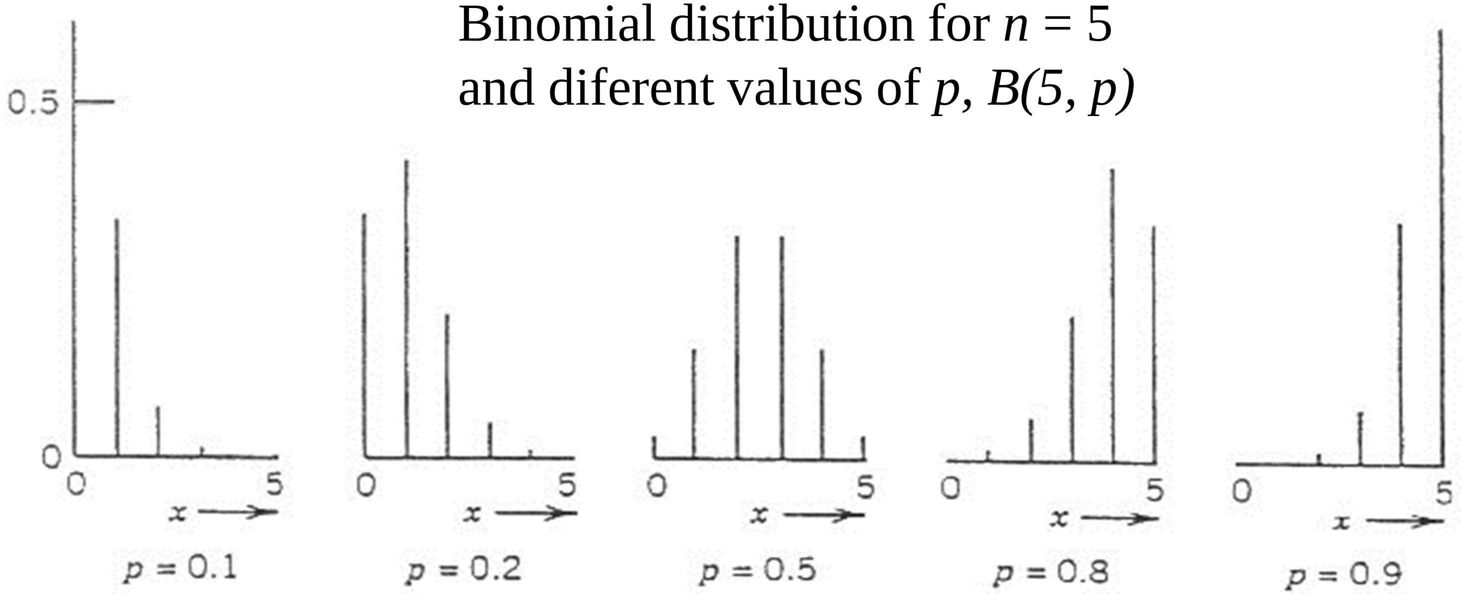


Binomial distribution

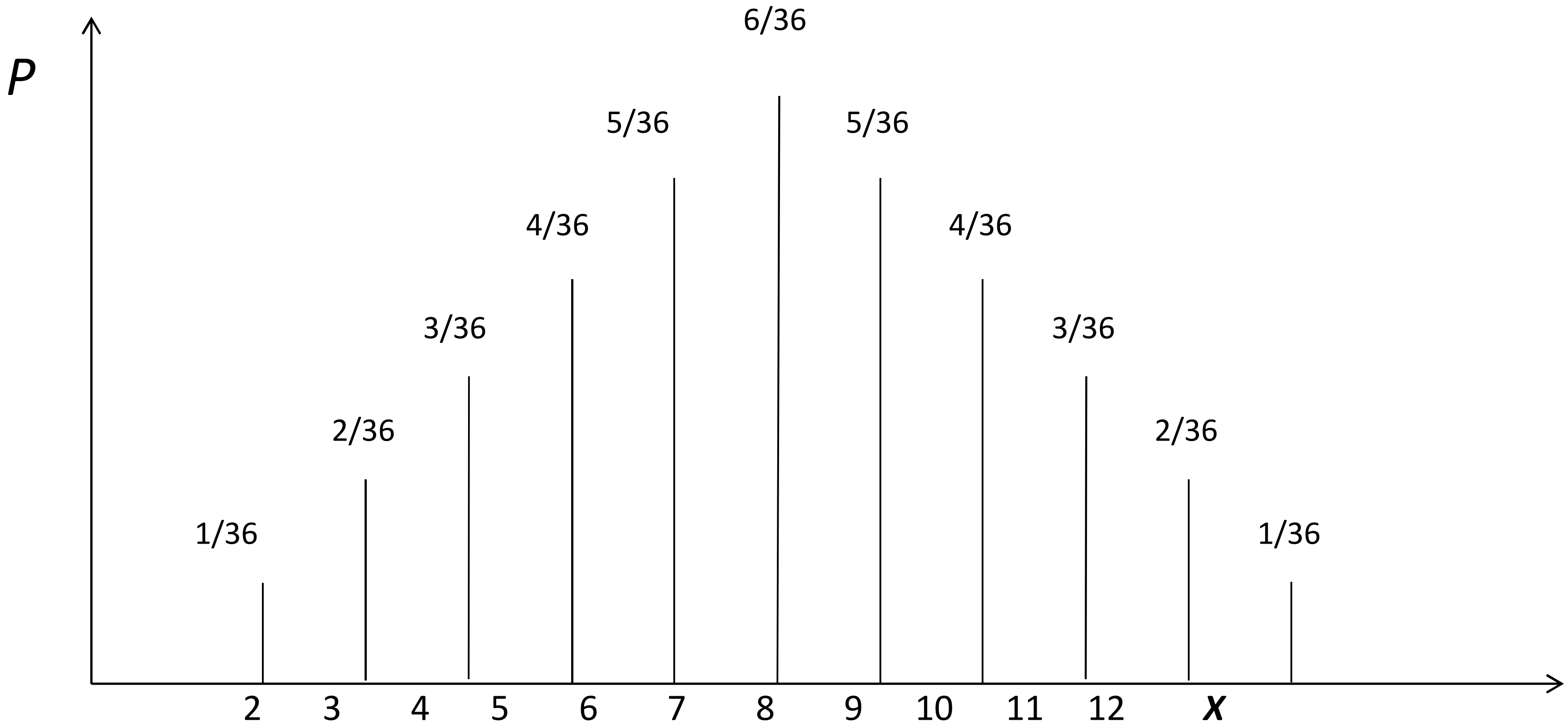
- In this case, the likelihood function is:

$$B(n,p) = p(x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

Binomial distribution for $n = 5$
and diferent values of p, $B(5, p)$



Binomial Distribution Example: two dice rolled



Proposed issues

- 30% of UCA students are myopic. If it gets to 20 students randomly, what is the probability that at most there are two myopic?
- A coin is thrown 10 times. What is the probability that they leave within 3 heads?
- A farmer plant 12 tomato plants. On average, 15% die the first winter. Calculate the probability that more than one die this winter
- Light bulbs are packaged in boxes 20. One light bulb of 10 is defective medium. What is the probability that a box has two defective bulbs?

Solutions : (1) 0.0355 (2) 0.0547 (3) 0.5565 (4) 0.285

Poisson distribution

- Gets the probability that an event occurring x times in a certain time period, knowing that the average number of occurring, λ
- Since x is the occurrences number, $x = 0,1,2,3, ...$
- The likelihood function is:

$$\boxed{p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0,1,2,\ldots}$$

- Example: Check photon detector

Poisson = binomial limit

- When a binomial distribution the number of trials (n) is large and the probability of success is small, the binomial distribution converges to the Poisson distribution ($\lambda = n \cdot p$)

$$p(x) = \frac{n!}{x! \cdot (n-x)!} \cdot p^x \cdot (1-p)^{n-x} \qquad \text{tomando } \lambda = n \cdot p$$

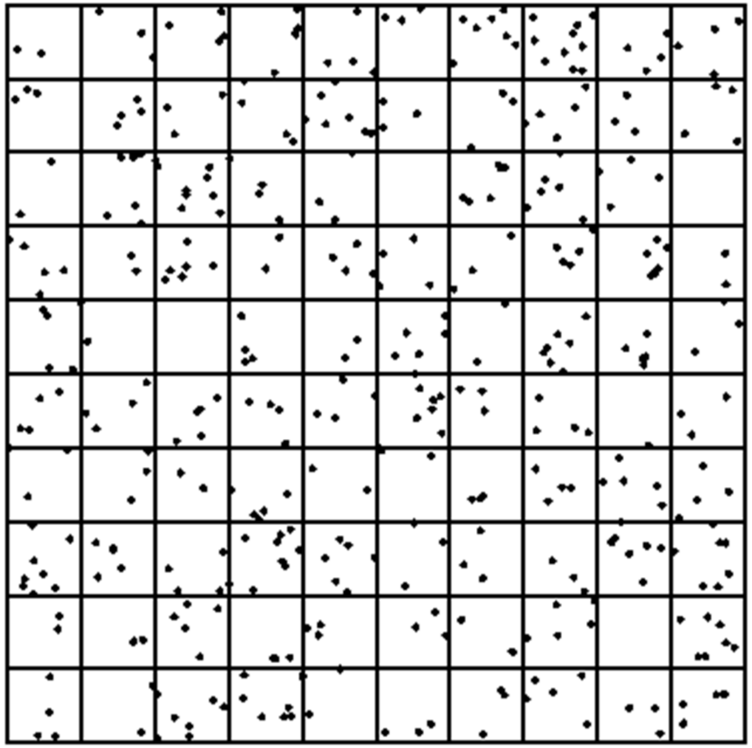
$$\lim_{n\to\infty} p(x) = \lim_{n\to\infty} \frac{n(n-1)(n-x+1)}{x!} \cdot \frac{\lambda^x}{n^x} \left(1 - \frac{\lambda}{n}\right)^n \cdot \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$$\lim_{n\to\infty} p(x) = \lim_{n\to\infty} \frac{n(n-1)(n-x+1)}{n^x} \cdot \frac{\lambda^x}{x!} \left(1 - \frac{\lambda}{n}\right)^n \cdot \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$$\lim_{n\to\infty} p(x) = 1 \cdot \frac{\lambda^x}{x!} e^{-\lambda} \cdot 1 = \frac{\lambda^x}{x!} e^{-\lambda}$$

Example: Bombs over London in World War II (Feller)

400 bombs



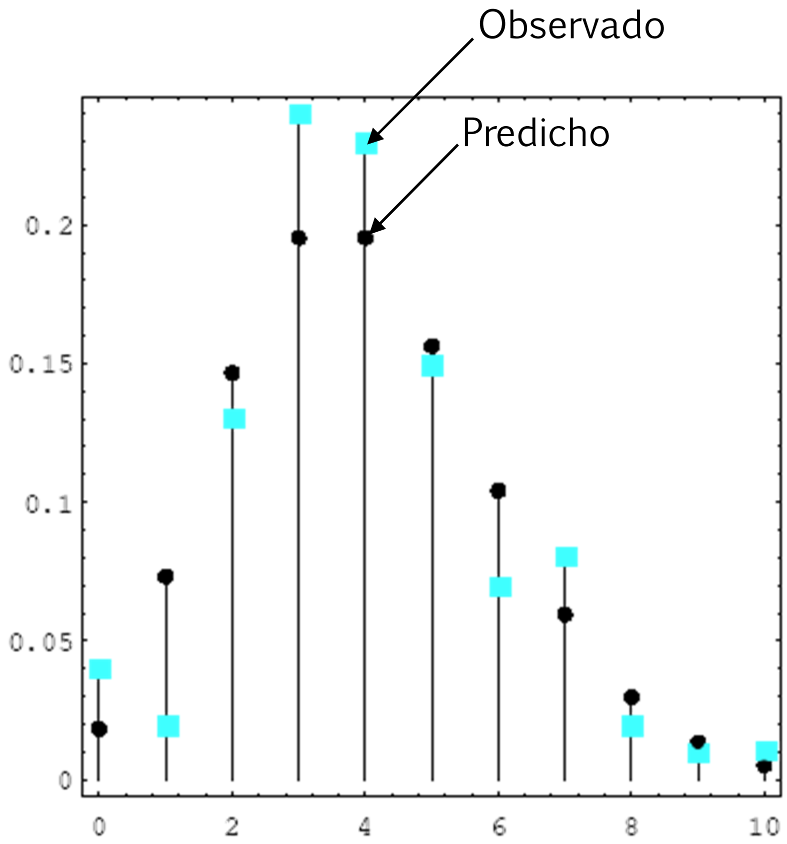
10 x 10

Example: Bombs over London in World War II (Feller)

- Suppose you lived in one of 100 blocks shown in the bottom graph
- The probability that a bomb was dropped on your block was $p = 1/100$
- As bombs fell 400, we can understand the number of hits on your block like the number of successes in Bernoulli experiment with $n = 400$ and $p = 1/100$
- We can use a Poisson with $\lambda = n * p = 400 * 1/100 = 4$.

Example: Bombs over London in World War II (Feller)

$$p(x) = \frac{e^{-4} 4^x}{x!}$$



Proposed issues

1. A secretary makes an average of two errors per page. How likely is it that you write a page without any error?
2. A computer has a "fall" every other day one average. What is the probability that there are 2 falls within a week?
3. They are selling toys with a mean number of failures 8. What is the probability that buying a toy with a single failure?

Solutions : (1) 0.135 (2)0.185 (3) 0.0027

Other distributions: multinomial

- When there are more than two possible events (A1, A2, A3 ...) with probabilities p1, p2, p3 ... constant and such that:

$$\sum_i p_i = 1$$

$$\boxed{p(x_1, x_2, x_3 \ldots) = \frac{n!}{x_1! x_2! x_3! ..} p_1^{x_1} . p_2^{x_2} . p_3^{x_3} ....}$$

Other distributions: Geometric

- It consists of repeating a Bernoulli experiment until the first success
- We define the random variable X as the number of failures until the first success is obtained

$$f(x) = (1-p)^x p \qquad x = 0,1,2,\ldots$$

$$F(n) = \sum_{x=0}^{n} (1-p)^x p = 1-(1-p)^{n+1}$$

- Example: either cross = success, how many times do I have to flip a coin to get a cross

Other distributions: negative binomial

- Bernoulli experiment repeated until the r-th success, the number of failures until the r-th success is obtained following the negative binomial distribution.

$$BN(r,p) = P(X = x) = \binom{x+r-1}{x} p^r (1-p)^x,$$

$$x = 0,1,2,\ \ldots$$

- Example: flipping a coin to get 3 heads

Other distributions: negative binomial

- The negative binomial distribution can also be defined as the number of testing until the *x* appearance of *r* successes.
- As the number of evidence x, in this case, counts both successes and failures as this definition would be:

$$BN(r,p) = P(X = x) = \binom{x+r-1}{x} p^r (1-p)^x,$$

$$x = 0,1,2,\ldots$$

- Example: flipping a coin to get 3 heads

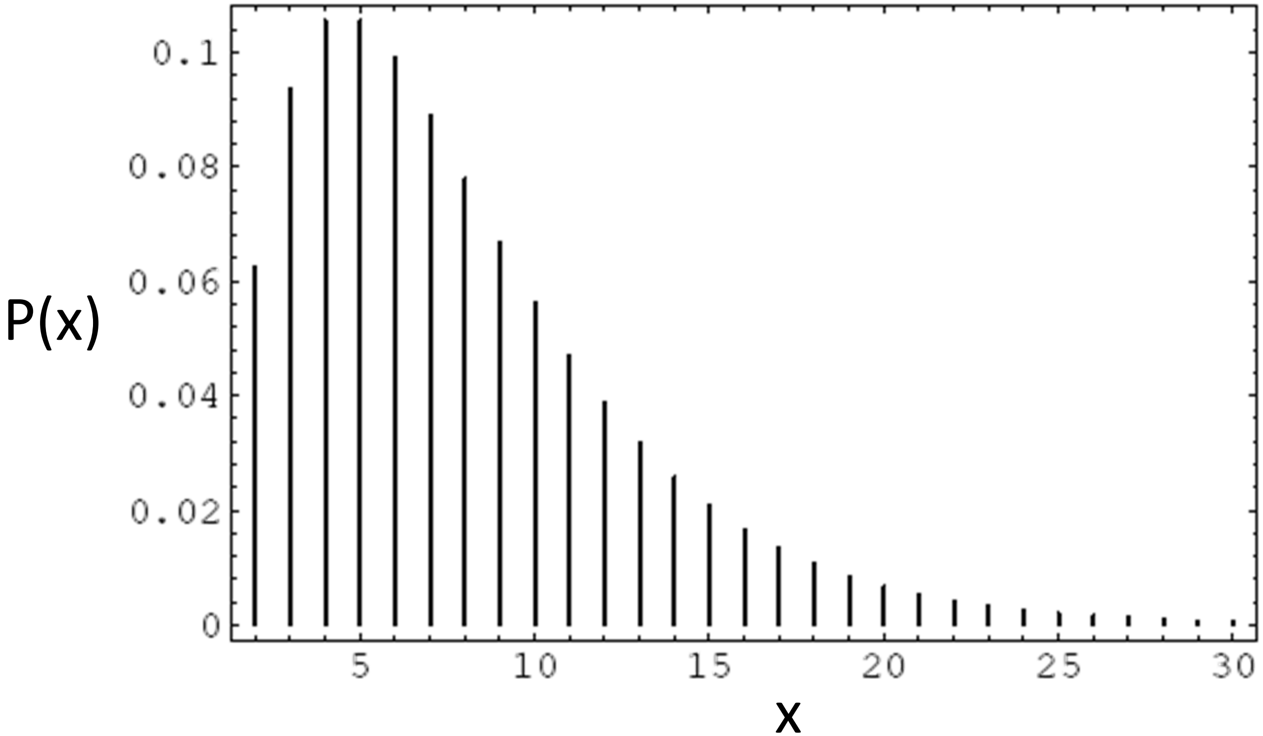
Example

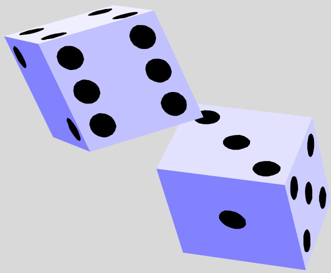
- We have a trick coin with probability of heads equal to p = 0.25. The launch until we get 2 sides. The distribution of the number of pitches x is:

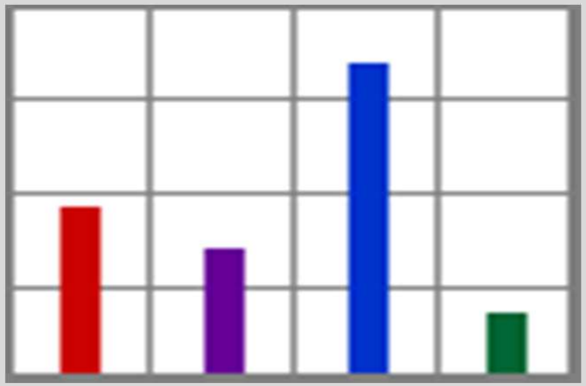
$$BN(r=2, p=0.25) = P(X=x) = \binom{x-1}{2-1} 0.25^2 (1-0.25)^{x-2},$$

$$x = 2,3,4,\ldots$$

Example







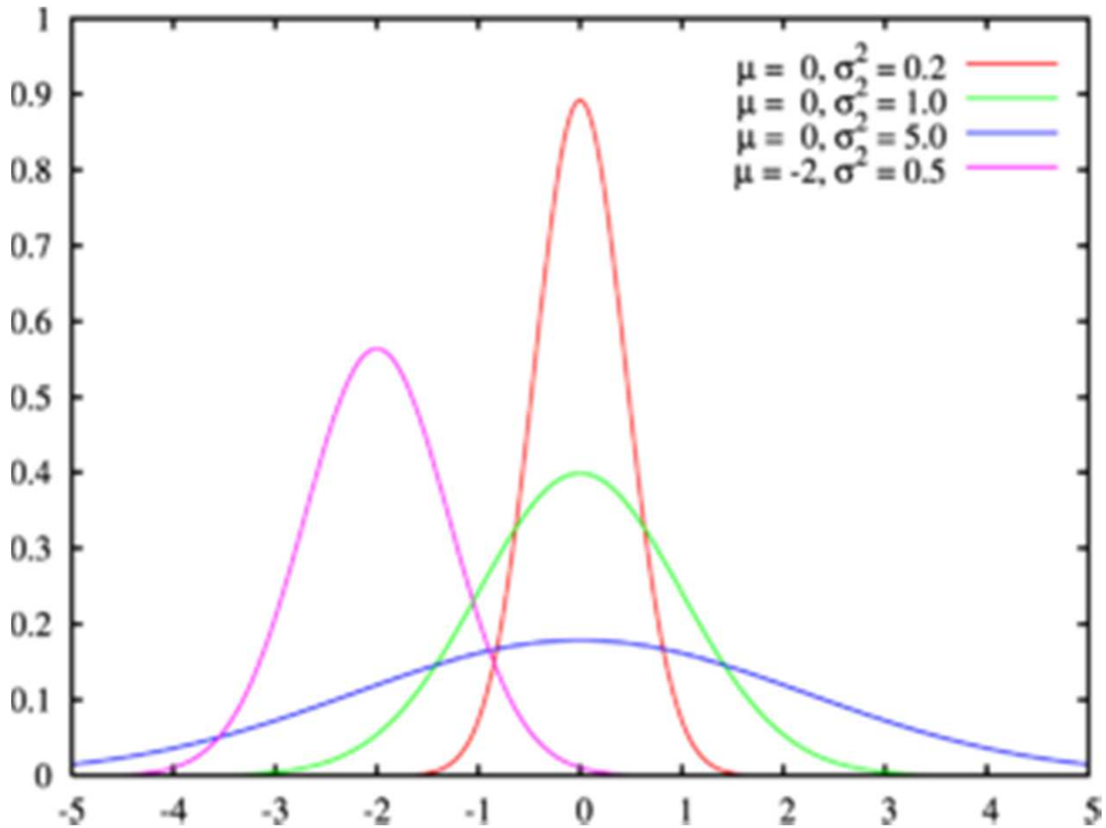
FREQUENTLY CONTINUOUS DISTRIBUTIONS

Normal distribution

- Normal, also called Gaussian or Gaussian
- Is the probability distribution that most often appears mainly because there are so many variables associated with natural phenomena that are modeled normal
  - Morphological characters of individuals
  - Physiological characteristics and the effect of a drug
  - Sociological characters consumption of a certain product by the same group of individuals
  - Psychological traits such as IQ
  - Noise level in Telecommunications
  - Errors in measuring magnitudes in experiments
  - Sample statistics such as the mean values
- It is also other distributions limit

Normal distribution: bell shape

- Two factors, the point where it is centered (mean) and the width of the hood (std. Standard)



Normal probability density function

- The probability density function is mathematically defined as:

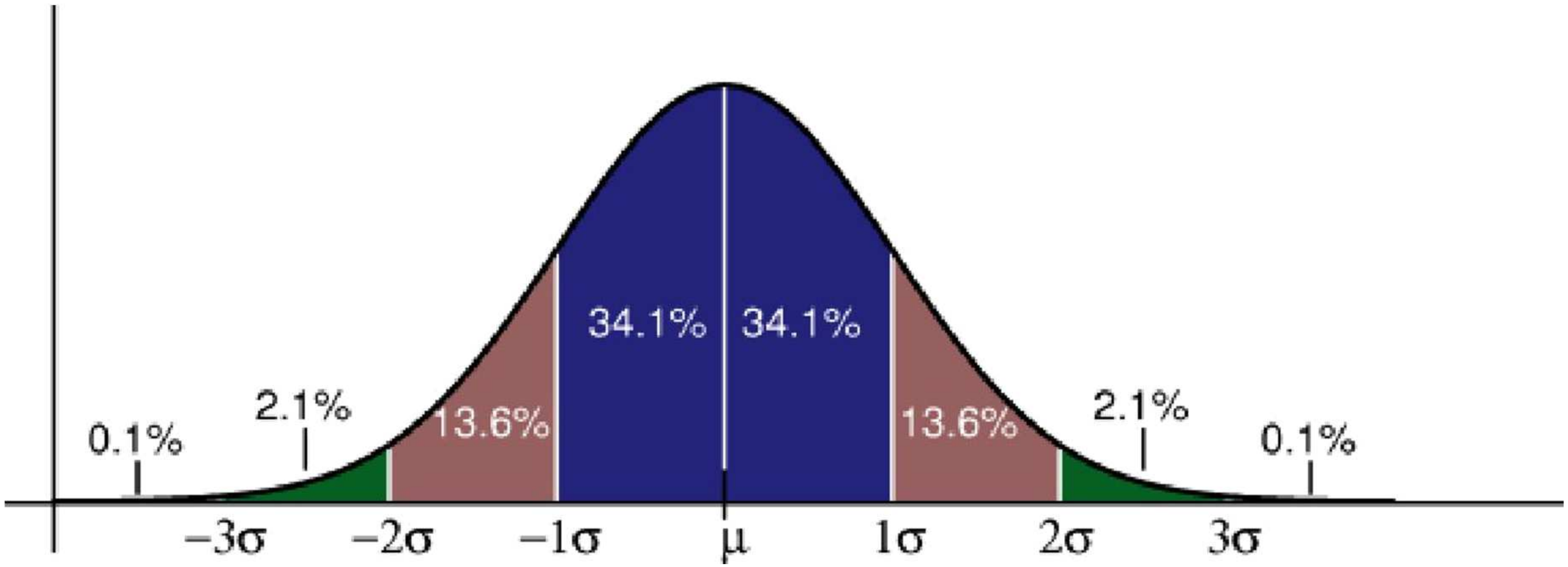
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ (mu) is the mean and σ (sigma) is the standard deviation

- Sigma squared (σ^2) is called the variance.

68-95-99.7 Rule

- Almost all data is within 3 standard deviations (3σ) of the mean (μ)

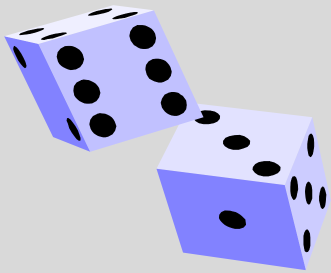


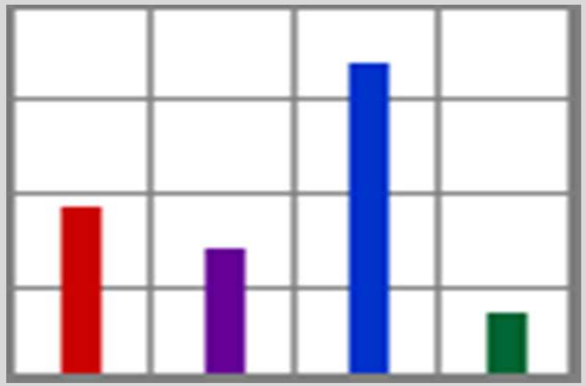
Normal distributions characterization

- If $\mu = 0$ and $\sigma = 1$, standard normal distribution.
- Given a normal random variable X with mean μ and standard deviation σ, if we define another random variable

$$Z = \frac{X - \mu}{\sigma}$$

then the random variable Z have a standard normal distribution





DRAWING FROM VARIOUS STATISTICAL DISTRIBUTIONS

Normal distribution

```
close all
N = 500;          % Numero de puntos generados
m = 3;            % Media de la distribucion
s = 2;            % Desv. tipica de la distribucion

x=randn(1,N);       % Datos
x=s*x+m;

I = m-3*s:s/5:m+3*s;        % Curva pdf gaussiana
h=normpdf(I,m,s);

p=hist(x,I);                % Histograma obtenido a
 partir de x
p=p/N*sum(h);               % Cambio de escala

bar(I,p);hold on;
plot(I,h,'r','LineWidth',3);hold off;
```

Normal distribution N(0,1)

```
close all
suma = randn(1,10000);

intervalo = -5:0.1:5;
a1=hist(suma,intervalo); a1=a1/sum(a1);
a2=normpdf(intervalo,0,1); a2=a2/sum(a2);

plot(intervalo,a1);hold on;
plot(intervalo,a2,'r');hold on;
```

Normal distribution N(μ,σ)

```
close all
m = 3;
s = 2;
suma = m + s*randn(1,10000);

intervalo = -5*s:s/10:5*s;
a1=hist(suma,intervalo); a1=a1/sum(a1);
a2=normpdf(intervalo,m,s); a2=a2/sum(a2);

plot(intervalo,a1);hold on;
plot(intervalo,a2,'r');hold on;
```

Drawing of a distribution χ^2 (N) from N distributions N (0,1)

```
close all
% La suma de N normales N(0,1) al cuadrado
% es una chi2 con N grados de libertad
%
 ===================================================
 ==
N = 10;
suma=sum(randn(N,10000).^2);

intervalo = 0:0.1:30;
a1=hist(suma,intervalo); a1=a1/sum(a1);
a2=chi2pdf(intervalo,N); a2=a2/sum(a2);

plot(intervalo,a1);hold on;
plot(intervalo,a2,'r');hold on;
```

Drawing of a distribution χ^2 (N) from N distributions N $(0,\sigma)$

```
% La suma de N normales N(0,v) es una chi2(N)
  %             Y = (v*X1)^2 + (v*X2)^2 + (v*X3)^2
  %             => Y/(v*v) es una chi2(3)
  %
=====================================================
=====
  close all
  N = 10;
  v = 2;
  suma=sum ((v*randn(N,10000)).^2);
  suma = suma/(v*v);

  intervalo = 0:0.1:30;
  a1=hist(suma,intervalo);
  a1=a1/sum(a1);
  a2=chi2pdf(intervalo,N);
  a2=a2/sum(a2);

  plot(intervalo,a1);hold on;
  plot(intervalo,a2,'r');hold on;
```

Drawing of a distribution χ^2 (N) from N distributions N (μ,σ)

```
% La suma de N normales N(mu,v) es una chi2(N) no centrada
con
  %         parametro delta = N*mu*mu/(v*v)
  %            Y = (v*X1+mu)^2 + (v*X2+mu)^2 + (v*X3+mu)^2
=>
  %            Y/(v*v) es una ncx2 con 3 grados de libertad
y
  %                                 con delta=3*mu*mu/v/v
  %
===========================================================
  N = 10;
  mu = 4;
  v  = 2;
  suma = sum((mu+v*randn(N,10000)).^2);
  suma=suma/(v*v);

  intervalo = 0:400;
  a1=hist(suma,intervalo);
  a1=a1/sum(a1);
  a2=ncx2pdf(intervalo,N,N*mu^2/(v*v));
  a2=a2/sum(a2);

  plot(intervalo,a1);hold on;
  plot(intervalo,a2,'r');hold on;
```

Drawing a distribution F(m, n)

```
% La suma de :
%          (N normales al cuadrado dividido por N) /
%          (M normales al cuadrado dividido por M)
%          es una F con M y N grados de libertad
%
%              (X1^2 + X2^2 + X3^2)/3
%           Y= ----------------------  => Y es una
 F(3,2)
%                   (X4^2 + X5^2)/2
N = 50;
M = 5;
suma1=sum(randn(N,10000).^2);
suma2=sum(randn(M,10000).^2);
suma=(suma1/N)./(suma2./M);

intervalo = 0:0.1:10;
a1=hist(suma,intervalo);   a1=a1/sum(a1);
a2=fpdf(intervalo,N,M);   a2=a2/sum(a2);

plot(intervalo,a1);hold on;
plot(intervalo,a2,'r');hold on;
```

Drawing a t-Student

```
% Si x y s son la media y desv. standard de una muestra de tamaño n
% obtenida de una N(mu,sigmacuadrado=n), entonces (x-mu)/s tiene una
% distribucion t de Student con n-1 grados de libertad

close all
N = 20000;      % Numero de puntos generados
n = 5;       % Tamaño de la muestra

datos = sqrt(n) * randn(n,N);
x = mean(datos);
s = std(datos);

I = -5:0.1:5;              % Curva pdf

h1=tpdf(I,n-1);

h2=normpdf(I,0,1);

p=hist(x,I);           % Histograma t-Student obtenido a partir de x
p=p/N*sum(h);          % Cambio de escala

bar(I,p);hold on;
plot(I,h1,'r','LineWidth',3);
plot(I,h2,'g','LineWidth',3);
hold off;
title('Rojo = t-Student , Verde=Normal')
```

Drawing a χ^2 (N) for passage to the limit (M $\rightarrow\infty$) of

```
% El limite, cuando M tiende a infinito, de la suma de :
%          (N normales al cuadrado dividido por N) /
%          (M normales al cuadrado dividido por M),
%          todo ello multiplicado por N
%          cuando M tiende a infinito
%          es una chi2 con N grados de libertad
%
%              lim        (X1^2 + X2^2 + X3^2)/3
%           Y=           -------------------------  => Y es una chi2
 con 3 grados
%              M->Inf    (X4^2 + X5^2 + ...XM^2)/M

N = 5;
M = 500;  % Aproximacion a Infinito
suma1=sum(randn(N,10000).^2);
suma2=sum(randn(M,10000).^2);
suma = N*(suma1/N)./(suma2./M);

intervalo = 0:0.1:20;
a1=hist(suma,intervalo);
a1=a1/sum(a1);
a2=chi2pdf(intervalo,N);
a2=a2/sum(a2);

plot(intervalo,a1);hold on;
plot(intervalo,a2,'r');hold on;
```

Drawing a χ^2 (N, δ) not center

```
% La suma de N normales al cuadrado, de media mu,
  %          es una chi2 no central con N grados de libertad,
 y
  %          parametro delta = N * mu * mu
  %
  %             Y = (X1+mu)^2 + (X2+mu)^2 + (X3+mu)^2 =>
  %             => Y es una ncx2 con 3 grados de lib. y delta
 = 3*mu*mu

 %============================================================
 ========
  N = 10;
  mu = 4;
  suma = sum((mu+randn(N,10000)).^2);

  intervalo = 0:400;
  a1=hist(suma,intervalo);
  a1=a1/sum(a1);
  a2=ncx2pdf(intervalo,N,N*mu^2);
  a2=a2/sum(a2);

  plot(intervalo,a1);hold on;
  plot(intervalo,a2,'r');hold on;
```

Drawing a F(m,n, δ) not cente

```
% La suma de :
   %         (N normales al cuadrado de media mu1 dividido por N) /
   %         (M normales al cuadrado dividido por M),
   %         es una F no central con M y N grados de libertad, y delta=N*mu1^2
   %
   %                 (X1^2 + X2^2 + X3^2)/3
   %          Y=  -------------------------
   %               (Y1^2 + Y2^2 + ...YM^2)/M
   %
   %          => Y es una ncf con 3 y M grados de libertad, y delta = 3*M*M,
   %
  ==============================================================================
   N = 3;
   M = 10;
   mu1=2;
   suma1=sum((mu1+randn(N,10000)).^2);
   suma2=sum(randn(M,10000).^2);
   suma = (suma1/N)./(suma2./M);

   intervalo = 0:0.1:25;
   a1=hist(suma,intervalo);
   a1=a1/sum(a1);
   a2=ncfpdf(intervalo,N,M,N*mu1*mu1);
   a2=a2/sum(a2);

   plot(intervalo,a1);hold on;
   plot(intervalo,a2,'r');hold on;
```

Drawing a $\chi^2(N, \delta)$ not centered by passage to the limit ($M \rightarrow \infty$) of F (N, M, δ) not centered

```
% El limite, cuando M tiende a infinito, de la suma de :
   %         (N normales al cuadrado de media mu1 y varia v1 dividido por N) /
   %         (M normales al cuadrado dividido por M),
   %         todo ello multiplicado por N
   %         cuando M tiende a infinito
   %         es una "ncx2" con M y N grados de libertad, y delta=N*mu1^2
   %
   %             lim          (X1^2 + X2^2 + X3^2)/3
   %          Y=             -------------------------
   %             M->Inf      (X4^2 + X5^2 + ...XM^2)/M
   %
   %          => Y es una ncx2 con 3 grados de libertad, y delta=3*mu1*mu1,
   %
   %
  =============================================================================
   N = 3;
   M = 500;  % Aproximacion a infinito
   mu1=2;
   suma1=sum((mu1+randn(N,10000)).^2 );
   suma2=sum(randn(M,10000).^2);
   suma = (suma1/N)./(suma2./M) * N;

   intervalo = 0:50;
   a1=hist(suma,intervalo);
   a1=a1/sum(a1);
   a2=ncx2pdf(intervalo,N,N*mu1*mu1);
   a2=a2/sum(a2);

   plot(intervalo,a1);hold on;
   plot(intervalo,a2,'r');hold on;
```

Step to limit the ncf = < ncx

```
% demo 9: El limite, cuando M tiende a infinito, de la suma de :
  %         (N normales al cuadrado de media mu1 y varia v1 dividido por N) /
  %         (M normales al cuadrado de media mu2 y varia v2),
  %         todo ello multiplicado por N
  %         cuando M tiende a infinito
  %         es una "ncx2" con M y N grados de libertad, y delta=N*mu1^2
  %
  %             lim        ((mu1+v1*X1)^2 + (mu1+v1*X2)^2 + (mu1+v1*X3)^2)/3
  %          Y=           -----------------------------------------------------
  %             M->Inf    ((mu2+v2*X4)^2 + (mu2+v2*X5)^2 + ...+(mu2+v2*XM)^2)/M
  %
  %           => Y es una ncx2 con 3 grados de libertad, y delta=3*mu1*mu1,
  %
  % =============================================================================

%========================================================
%    !!!!!! SOLO FUNCIONA SI mu2 ES IGUAL A 0 !!!!!!!!!
% =======================================================
  N = 3;
  M = 200;
  mu1=3;
  mu2=0;
  v1=2;
  v2=5;
  suma1=0;
  for i=1:N,
     suma1=suma1 + (mu1+v1*randn(1,10000)).^2;
  end;

  suma2=0;
  for i=1:M,
     suma2=suma2 + (mu2+v2*randn(1,10000)).^2;
  end;

  if mu2~=0,
     disp('Este caso aun no lo he resuelto');
  end;

  suma = (suma1/N)./(suma2./M) * N /v1/v1*v2*v2;

  intervalo = 0:0.25:25;
  a1=hist(suma,intervalo);
  a1=a1/sum(a1);
  a2=ncx2pdf(intervalo,N,N*mu1*mu1/v1/v1);
  a2=a2/sum(a2);

  plot(intervalo,a1);hold on;
  plot(intervalo,a2,'r');hold on;
```

Step to limit the ncf = < ncx

```
% demo 10: El limite, cuando M tiende a infinito, de la suma de :
   %         (N normales al cuadrado de media mu1 y varia v1 dividido por N) /
   %         (M normales al cuadrado de media mu2 y varia v2),
   %         todo ello multiplicado por N
   %         cuando M tiende a infinito
   %         es una "ncx2" con M y N grados de libertad, y delta=N*mu1^2
   %
   %             lim        ((mu1+v1*X1)^2 + (mu1+v1*X2)^2 + (mu1+v1*X3)^2)/3
   %          Y=           -----------------------------------------------------
   %             M->Inf    ((mu2+v2*X4)^2 + (mu2+v2*X5)^2 + ...+(mu2+v2*XM)^2)/M
   %
   %           => Y es una ncx2 con 3 grados de libertad, y delta=3*mu1*mu1,
   %
   % ============================================================================

   N = 3;
   M = 200;
   mu1=3;
   mu2=0.5;
   v1=12;
   v2=1;
   suma1=0;
   for i=1:N,
      suma1=suma1 + (mu1+v1*randn(1,10000)).^2;
   end;

   suma2=0;
   for i=1:M,
      suma2=suma2 + (mu2+v2*randn(1,10000)).^2;
   end;

   if mu2~=0,
      disp('Este caso aun no lo he resuelto');
   end;

   suma = (suma1/N)./(suma2./M) * N /v1/v1*v2*v2;

   intervalo = 0:0.25:25;
   a1=hist(suma,intervalo);
   a1=a1/sum(a1);
   a2=ncx2pdf(intervalo,N,N*((mu1/v1)^2+(v2/mu2)^2));
   a2=a2/sum(a2);

   plot(intervalo,a1);hold on;
   plot(intervalo,a2,'r');hold on;
```