SEMINAR 1: THE WEATHER AND MARKOV MODELS

Grado en Ingeniería Informática

Curso 2014 / 15

Objectives

- By the end of this lesson, students will:
  - **Define** what a **Markov Model** (MM) is
  - **Describe** the elements of **MM**
  - **Apply** MM to predict the weather

Introduction

- http://www.youtube.com/watch?v=VBOGxdLCMqE

Introduction

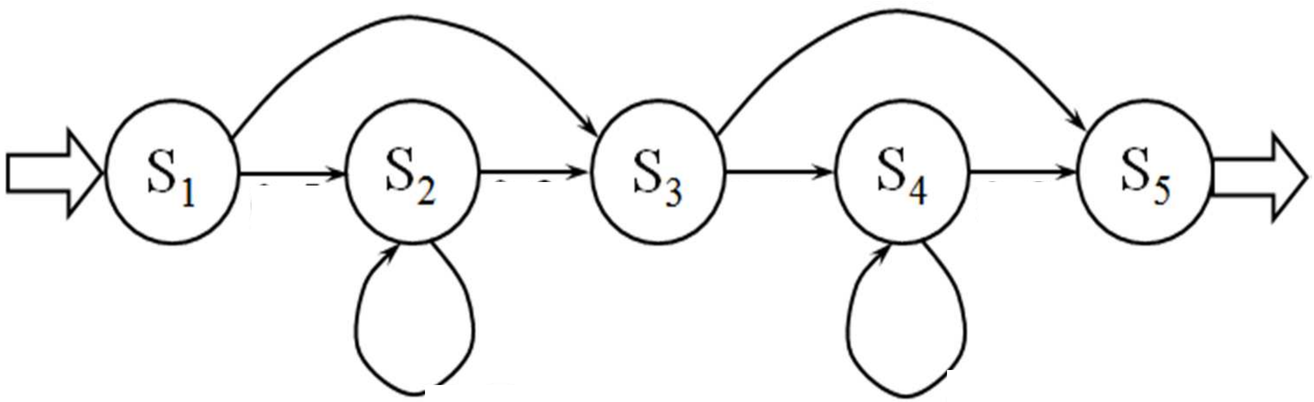
UCA Universidad de Cádiz

- Questions about the video:

1. What do you think a Markov Model is?
2. What is the key of Markov Models?
3. In your opinion, what are the applications of Markov Models?

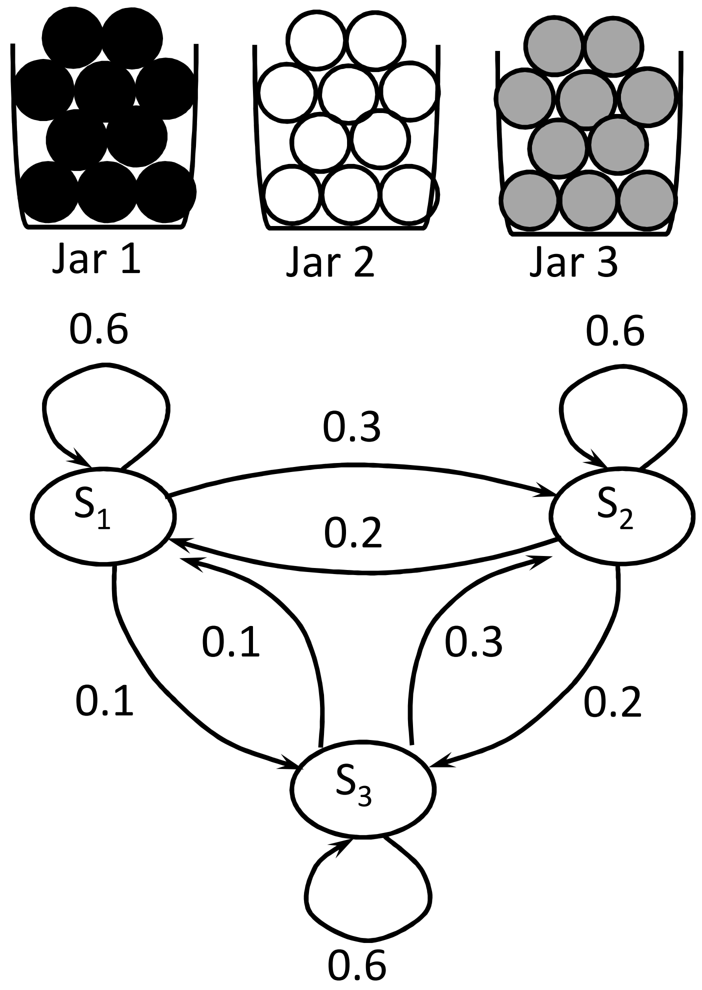
What is a Markov Model (MM)?

- A Markov Model or Markov chain is similar to a **finite-state automata**, with **probabilities of transitioning** from one state to another.



MM Elements

- States: $Q = \{1, 2, \ldots, N\}$
- Initial probabilities:
  $\pi_j = P[q_1 = j], 1 \le j \le N$
- Transition probabilities:
  $a_{ij} = P[q_t = j \,|q_{t-1} = i], 1 \le i, j \le N$
- Events: $E = \{e_1, e_2, \ldots, e_N\}$
- Clock: $t = \{1, 2, \ldots, T\}$



MM Elements

- S_1 = event$_1$ = black
  S_2 = event$_2$ = white
  S_3 = event$_3$ = grey

$$A = \{a_{ij}\} = \begin{pmatrix} .60 & .30 & .10 \\ .20 & .60 & .20 \\ .10 & .30 & .60 \end{pmatrix}$$

$\pi_1 = 0.33$
$\pi_2 = 0.33$
$\pi_3 = 0.33$

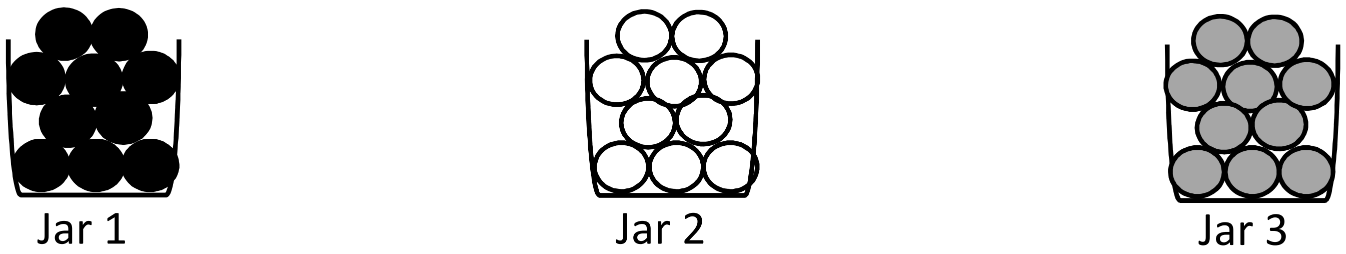
- What is probability of {grey, white, white, black, black, grey}?
  Obs. = {g, w, w, b, b, g}
  S = $\{S_3, S_2, S_2, S_1, S_1, S_3\}$
  time = {1, 2, 3, 4, 5, 6}

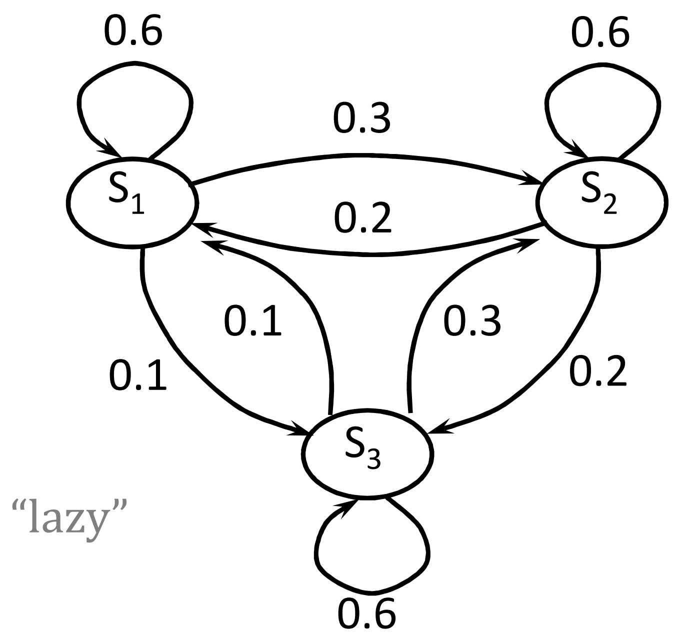
$$= P[S_3] \cdot P[S_2|S_3] \cdot P[S_2|S_2] \cdot P[S_1|S_2] \cdot P[S_1|S_1] \cdot P[S_3|S_1]$$

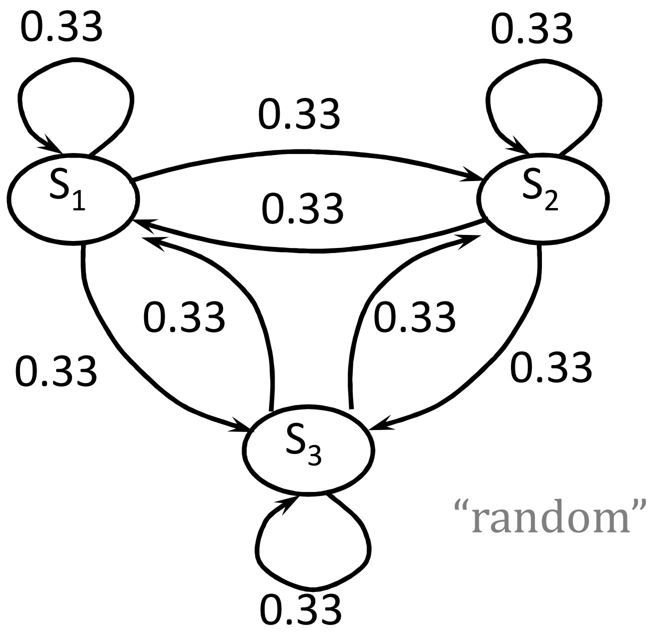
$$= 0.33 \cdot 0.3 \cdot 0.6 \cdot 0.2 \cdot 0.6 \cdot 0.1$$

$$= 0.0007128$$

MM Elements







- Same data, two different models...

MM Elements

What is probability of:

{w, g, b, b, w}

given each model ("lazy" and "random")?

S = $\{S_2, S_3, S_1, S_1, S_2\}$
time = {1, 2, 3, 4, 5}

"lazy"

$= P[S_2]\ P[S_3|S_2]\ P[S_1|S_3]\ P[S_1|S_1]\ P[S_2|S_1]$

$= 0.33 \cdot 0.2 \cdot 0.1 \cdot 0.6 \cdot 0.3$

$= 0.001188$

"random"

$= P[S_2]\ P[S_3|S_2]\ P[S_1|S_3]\ P[S_1|S_1]\ P[S_2|S_1]$

$= 0.33 \cdot 0.33 \cdot 0.33 \cdot 0.33 \cdot 0.33$

$= 0.003913$

{w, g, b, b, w} has greater probability if generated by "random."

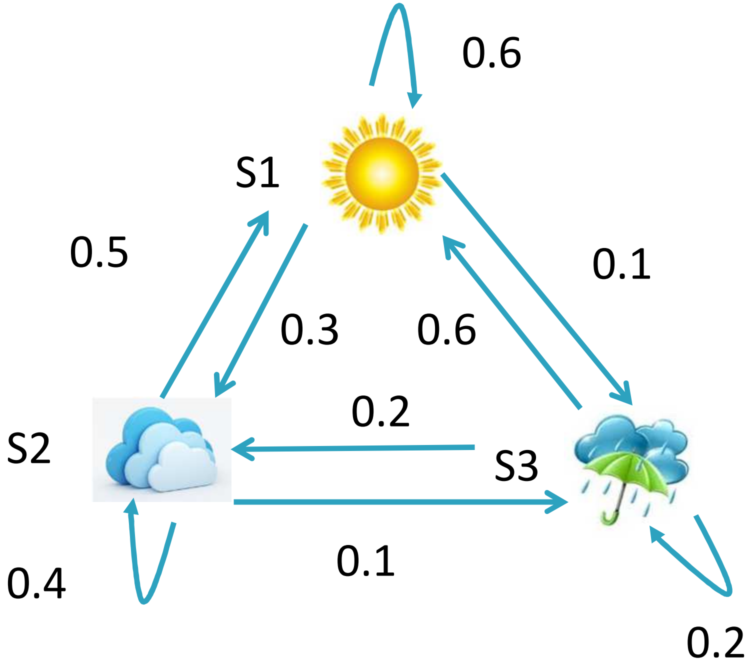
⇒"random" model more likely to generate {w, g, b, b, w}.

MM Elements

- Notes:
  - *Independence* is assumed between events that are separated by more than one time frame, when computing probability of sequence of events (for first-order model).
  - Given list of observations, I can determine exact state sequence.
    $\Rightarrow$ state sequence not *hidden.*
  - Each state associated with only one event (output).
  - Computing probability of a given observation and model is straightforward.
  - Given multiple Markov Models and an observation sequence, it's easy to determine the M.M. most likely to have generated the data.

The weather in Cádiz

Cádiz



$$A = \{a_{ij}\} = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.5 & 0.4 & 0.1 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$$

$$\pi_1 = 0.6 \quad \pi_2 = 0.3 \quad \pi_3 = 0.1$$

1 2 3 4 5 6 7 Days

Ejemplo: el Tiempo en londre

- Observations = {Rainy, Rainy, Rainy, Cloudy, Sunny, Cloudy, Rainy}
- State sequence = $\{S_1, S_1, S_1, S_2, S_3, S_2, S_1\}$
- Time = {1, 2, 3, 4, 5, 6, 7} (days)
- $P = P[S_1] \cdot P[S_1|S_1] \cdot P[S_1|S_1] \cdot P[S_2|S_1] \cdot P[S_3|S_2] \cdot P[S_2|S_3] \cdot P[S_1|S_2] = 0.6 \cdot 0.6 \cdot 0.6 \cdot 0.3 \cdot 0.1 \cdot 0.2 \cdot 0.5 = 6.4800e-004$