Example of a SAT problem resolution

1 Introduction

This problem was taken from http://www.udlap.mx/~leolopez/TabooSearch.htm and the solution has been adapted and written according to the taboo algorithm we have previously studied in this unit 2.

2 Problem definition

SAT (Satisfiability) problems aim at finding the best assignment for a number of variables in order to minimize C(x) function, restricted to some constraints which can penalize the potential solutions.

Let's assume the following Sat problem with 5 variables:

$F(x)=20\ x_1 + 25\ x_2 - 30\ x_3 - 45\ x_4 + 40\ x_5$ where $x_j = \{ 0,1 \}, j = 1,...,5$

Constraints:

$x_1 + x_2 - x_3 + x_4 + x_5 >= 1$

$x_1 + x_2 - x_4 + 2x_5 >= 2$

$-x_2 + x_4 + x_5 <= 1$

$x_2 + x_3 + x_5 <= 2$

Penalization: Each constraint violation costs:

- 70 (per each) for the two first constraints
- 100 (per each) for the two last constraints

The final cost to minimize is:

$$\mathbf{C(x) = F(x) + Penalization(x)}$$

Actions: Swap the value of each variable (0 or 1)

- There exists a taboo list per each variable of the function.
- The taboo tenure is set to 4 iterations
- Best solution found until now is kept

3 SAT Resolution using Taboo Search

From this initial state: x^0 (1, 0, 0, 0, 1) where the final cost is $C(x^0)= 60$

Initially: Initial solution: $x^0 = (1, 0, 0, 0, 1)$, $c(x^0)=60$ Taboo list = (0,0,0,0,0)

Best solution: $x^0 = (1, 0, 0, 0, 1)$, $c(x^0)=60$

Iteration 1: x^0 (1, 0, 0, 0, 1)

Successor states obtained from the current state

$m_1(x^0) : x_1 = 0$ x = (0,0,0,0,1) => C(x)= 40

$m_2(x^0): x_2=1$ x = (1,1,0,0,1) => C(x)= 85

$m_3(x^0): x_3=1$ x = (1,0,1,0,1) => C(x)= 30*

$m_4(x^0):x_4=1$ x = (1,0,0,1,1) => C(x)= 15 +100=115

$m_5(x^0):x_5=0$ x = (1,0,0,0,0) => C(x)= 20+70=90

- The best successor is x^1(1, 0, 1, 0, 1), $C(x^1) = 30$
- This option x^1 is not taboo:
  - Current solution**: x^1 = (1, 0, 1, 0, 1), $C(x^1)$ = 30**
  - Taboo list = **(0 0 4 0 0)** Keep the taboo tenure for variable 3
  - Best solution: $x^1 = (1,0,1,0,1)$, $C(x^1) = 30$

Iteration 2: $x^1 = (1,0,1,0,1)$

Successor states obtained from the current state

$m_1(x^1) : x_1 = 0$ x = (0,0,1,0,1) => C(x)= 10 + 70 = 80*

$m_2(x^1) : x_2 = 1$ x = (1,1,1,0,1) => C(x)= 55 + 100 = 155

$m_3(x^1) : x_3 = 0$ x = (1,0,0,0,1) => C(x)= 60 T

$m_4(x^1) : x_4 = 1$ x = (1,0,1,1,1) => C(x)= -15 +100= 85

$m_5(x^1) : x_5 = 0$ x = (1,0,1,0,0) => C(x)= -10 +140 =130

- Best successor $x^2 = (1, 0, 0, 0, 1)$, $C(x^2) = 60$
- x^2 is taboo and the aspiration criterion is not met
- Next best successor $x^2 = (0, 0, 1, 0, 1)$, $C(x^2) = 80$
- This option x^2 is not taboo:

- Solution $x^2 = (0, 0, 1, 0, 1)$, $C(x^2) = 80$
- Taboo list = **(4 0 3 0 0)** (tenure values must be decreased in each iteration)
- Best solution: $x^1 = (1,0,1,0,1)$, $C(x^1) = 30$

Iteration 3: $x^2 = (0, 0, 1, 0, 1)$

Successor states obtained from the current state

$m_1(x^2) : x_1 = 0$ x = (1,0,1,0,1) => C(x)= 30 T

$m_2(x^2) : x_2 = 1$ x = (0,1,1,0,1) => C(x)= 35 + 100=135

$m_3(x^2) : x_3 = 0$ x = (0,0,0,0,1) => C(x)= 40 T

$m_4(x^2) : x_4 = 1$ x = (0,0,1,1,1) => C(x)= -35 +170= 135

$m_5(x^2) : x_5 = 0$ x = (0,0,1,0,0) => C(x)= -30 +140 =110*

- First successor is taboo and it does not improve the Best
- Next best successor is taboo and it does not improve the Best
- Next best successor $x^3=(0,0,1,0,0)$, $C(x^3)=110$
- This option is not taboo:
  - Current solution $x^3=(0,0,1,0,0)$, $C(x^3)=110$
  - **Taboo list = (3, 0, 2, 0,4)**
  - Best solution: $x^1 = (1,0,1,0,1)$, $C(x^1) = 30$

Iteration 4: $x^3=(0,0,1,0,0)$,

Successor states obtained from the current state

$m_1(x^3) : x_1 = 1$ x = (1,0,1,0,0) => C(x)= 130 T

$m_2(x^3) : x_2 = 1$ x = (0,1,1,0,0) => C(x)= 35 + 100=135

$m_3(x^3) : x_3 = 0$ x = (0,0,0,0,0) => C(x)= 40 + 100 =140 T

$m_4(x^3) : x_4 = 1$ x = (0,0,1,1,0) => C(x)= 65

$m_5(x^3) : x_5 = 0$ x = (0,0,1,0,1) => C(x)=80 T

- Best successor $x^3=(0,0,1,1,0)$, $C(x^3)=65$
- This option is not taboo:
  - Current solution $x^4=(0,0,1,1,0)$, $C(x^3)=65$
  - **Taboo list = (2, 0, 1, 4,3)**
  - Best solution: $x^1 = (1,0,1,0,1)$, $C(x^1) = 30$

Iteration 5: x^4=(0,0,1,1,0),

Successor states obtained from the current state

$m_1(x^4) : x_1 = 1$ x = (1,0,1,1,0) => C(x)= 15 T

$m_2(x^4) : x_2 = 1$ x = (0,1,1,1,0) => C(x)= 20

$m_3(x^4) : x_3 = 0$ x = (0,0,0,1,0) => C(x)= 25

$m_4(x^4) : x_4 = 1$ x = (0,0,1,0,0) => C(x)= 110

$m_5(x^4) : x_5 = 0$ x = (0,0,1,1,1) => C(x)= 135

- Best successor x^5=(1,0,1,1,0), C(x^5)=15
- This option is taboo and Improve the best current solution (Aspiration criterion)
    - Current solution x^5=(1,0,1,1,0), C(x^5)=15
    - Best solution: x^5 = (1,0,1,1,0), C(x^5) = 15
    - **Taboo list = (4, 0, 0, 3,2)**

Iteration 6: x^5 =(1 0 1 1 0)

Successor states obtained from the current state

$m_1(x^5) : x_1 = 0$ x = (0,0,1,1,0) => C(x)= 65 T

$m_2(x^5) : x_2 = 1$ x = (1,1,1,1,0) => C(x)= 40 *

$m_3(x^5) : x_3 = 0$ x = (1,0,0,1,0) => C(x)= 45

$m_4(x^5) : x_4 = 0$ x = (1,0,1,0,0) => C(x)= 130 T

$m_5(x^5) : x_5 = 1$ x = (1,0,1,1,1) => C(x)= 85 T

- Best successor x^6=(1,1,1,1,0), C(x^6)=40
- This option is not taboo and it does not improve the best solution:
    - Current solution x^6=(1,1,1,1,0), C(x^6)=40
    - Best solution: x^5 = (1,0,1,1,0), C(x^5) = 15
    - **Taboo list = (3, 0, 0, 2,1)**

After 6 iterations the "optimal" solution is:

x* = (1,0,1,1,0) con C(x*)=15