Example of a SAT problem resolution

SAT problems aim at finding the best assignment for a number of variables in order to minimize C(x) function, restricted to some constraints which can penalize the potential solutions.

Let's assume the following Sat problem with 5 variables:

$F(x)=20 x_1 + 25 x_2 - 30 x_3 - 45 x_4 + 40 x_5$ where $x_j = \{ 0,1 \}, j = 1,...,5$

Constraints:

$x_1+x_2-x_3+x_4+x_5 >=1$

$x_1 + x_2 - x_4 +2x_5 >= 2$

$-x_2 + x_4 + x_5 <= 1$

$x_2+ x_3+ x_5 <= 2$

Penalization: Each constraint violation costs:

- 70 (per each) for the two first constraints
- 100 (per each) for the two last constraints

The final cost to minimize is:

$$\mathbf{C(x) = F(x) + Penalization(x)}$$

Actions: Swap the value of each variable (0 or 1)

- There exists a taboo list per each variable of the function.
- The taboo tenure is set to 4 iterations
- Best solution found until now is kept

3 SAT Resolution using Taboo Search

From this initial state: x^0 (1, 0, 0, 0, 1) where the final cost is $C(x^0)$= 60

Initially: Initial solution: $x^0 = (1, 0, 0, 0, 1)$, $c(x^0)=60$ Taboo list = (0,0,0,0,0)

Best solution: $x^0 = (1, 0, 0, 0, 1)$, $c(x^0)=60$

Iteration 1: Current state: x^0 (1, 0, 0, 0, 1)

Successor states obtained from the current state

$m_1(x^0) : x_1 = 0$	$x = (0,0,0,0,1)$	=>	C(x)= 40
$m_2(x^0): x_2=1$	$x = (1,1,0,0,1)$	=>	C(x)= 85
$m_3(x^0): x_3=1$	$x = (1,0,1,0,1)$	=>	C(x)= 30*
$m_4(x^0):x_4=1$	$x = (1,0,0,1,1)$	=>	C(x)= 15 +100=115
$m_5(x^0):x_5=0$	$x = (1,0,0,0,0)$	=>	C(x)= 20+70=90

- The best successor is x^1(1, 0, 1, 0, 1), $C(x^1) = 30$
- This option x^1 is not taboo:
  - Current solution**: x^1 = (1, 0, 1, 0, 1), $C(x^1)$ = 30**
  - Taboo list **= (0 0 4 0 0)** Keep the taboo tenure for variable 3
  - Best solution: $x^1 = (1,0,1,0,1)$, $C(x^1) = 30$

Iteration 2: $x^1 = (1,0,1,0,1)$

Successor states obtained from the current state